Price Stability and Monetary Policy Effectiveness when Nominal Interest Rates are Bounded at Zero[*,†]

Günter Coenen
European Central Bank

Athanasios Orphanides
Board of Governors of the Federal Reserve System

Volker Wieland
Goethe University of Frankfurt and CEPR

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Abstract

This paper employs stochastic simulations of a small structural rational expectations model to investigate the consequences of the zero bound on nominal interest rates. We find that if the economy is subject to stochastic shocks similar in magnitude to those experienced in the U.S. over the 1980s and 1990s, the consequences of the zero bound are negligible for target inflation rates as low as 2 percent. However, the effects of the constraint are non-linear with respect to the inflation target and produce a quantitatively significant deterioration of the performance of the economy with targets between 0 and 1 percent. The variability of output increases significantly and that of inflation also rises somewhat. Also, we show that the asymmetry of the policy ineffectiveness induced by the zero bound generates a non-vertical long-run Phillips curve. Output falls increasingly short of potential with lower inflation targets.

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1 Introduction

There is fairly widespread consensus among macroeconomists that the primary long-term objective of monetary policy ought to be a stable currency. Studies evaluating the costs of inflation have long established the desirability of avoiding not only high but even moderate inflation.[1] However, there is still a serious debate on whether the optimal average rate of inflation is low and positive, zero, or even moderately negative.[2] An important issue in this debate concerns the reduced ability to conduct effective countercyclical monetary policy when inflation is low. As pointed out by Summers (1991), if the economy is faced with a recession when inflation is zero, the monetary authority is constrained in its ability to engineer a negative short-term real interest rate to damp the output loss. This constraint reflects the fact that the nominal short-term interest rate cannot be lowered below zero—the zero interest rate bound.[3]

This constraint would be of no relevance in the steady state of a non-stochastic model economy. In an equilibrium with zero inflation, the short-term nominal interest rate would always equal the equilibrium real rate. Stabilization of the economy in a stochastic environment, however, presupposes monetary control which leads to fluctuations in the short-run nominal interest rate. Under these circumstances, the non-negativity constraint on nominal interest rates may occasionally be binding and so may influence the performance of the economy. This bound is more likely to be reached, the lower the average rate of inflation and

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[1]Fischer and Modigliani (1978), Fischer (1981), and more recently Driffill et al. (1990) and Fischer (1994), provide a detailed accounting of the costs of inflation. An early analysis of the costs of both inflation and deflation is due to Keynes (1923).

[2]The important contributions by Tobin (1965) and Friedman (1969) provided arguments in favor of inflation and deflation, respectively. But theoretical arguments alone cannot provide a resolution. The survey of the monetary growth literature by Orphanides and Solow (1990) suggests that equally plausible assumptions yield conflicting conclusions regarding the optimal rate of inflation. Similarly, recent empirical investigations suggest a lack of consensus. Cross-country studies confirm the cost of high average inflation on growth but find no robust evidence at low levels of inflation. (See Sarel, 1996, and Clark, 1997.) Judson and Orphanides (1999) find that the volatility rather than the level of inflation may be detrimental to growth at low levels of inflation. Feldstein (1997) identifies substantial benefits from zero inflation due to inefficiencies in the tax code. Akerlof, Dickens and Perry (1996), however, estimate large costs due to downward wage rigidities.

[3]The argument has its roots in Hicks's (1937) interpretation of the Keynesian liquidity trap. Hicks (1967) identified the question regarding "the effectiveness of monetary policy in engineering recovery from a slump" as the key short-run concern arising from the trap (p. 57).

the greater the variability of the nominal interest rate. In this context, "inflation greases the wheels of monetary policy," as Fischer (1996) points out (p. 19). The experience of the Japanese economy that has been at the zero bound over the past several years and the uncomfortable resemblance of this experience to the U.S. economy during the 1930s serve as evidence that the zero bound presents a challenge of significant practical importance.

The purpose of this paper is to conduct a systematic empirical evaluation of the zero bound constraint in a stochastic environment and assess the quantitative importance of this constraint for the performance of alternative monetary policy rules. Recent quantitative evaluations of policy rules suggest that rules that are very effective in stabilizing output and inflation do indeed entail substantial variability in the short-term nominal interest rate. (See Taylor, 1999.) Most often, however, the simulated models are linear and neutral to the average rate of inflation and abstract from the zero bound. Alternative policy rules are then evaluated based on their performance in terms of the variability of output and inflation they induce in such models. This approach to policy evaluation is appropriate with a high average rate of inflation when the non-negativity constraint on nominal interest rates would be unlikely to bind. However, since policy is not only concerned with stabilizing output and inflation but also with maintaining a low average inflation rate, evaluation of the impact of the zero bound on economic performance is important. To the extent that both inflation and deflation hamper economic performance and are otherwise equally undesirable, the zero bound constraint effectively renders the risks of deviating from an inflation rate of zero asymmetric. As Chairman Greenspan noted recently, "... deflation can be detrimental for reasons that go beyond those that are also associated with inflation. Nominal interest rates are bounded at zero, hence deflation raises the possibility of potentially significant increases in real interest rates." (From *Problems of Price Measurement,* remarks at the Annual Meeting of the American Economic Association and the American Finance Association, Chicago, Illinois, Jan 3, 1998.)

Efforts to evaluate the quantitative importance of the zero bound have been hampered by the nonlinearity it introduces a nonlinearity in otherwise linear models. In the context

of policy rule evaluations, Rotemberg and Woodford (1997, 1999) indirectly address the constraint by penalizing policies resulting in exceedingly variable nominal interest rates. They show that such constrained optimal policies significantly differ from the optimal rules that ignore the constraint. A first assessment of the effect of the zero bound that explicitly introduces this nonlinearity in a small linear model is provided by Fuhrer and Madigan (1997). Their results, based on a set of deterministic simulations, suggest that the reduced policy effectiveness at low inflation rates may have a modest effect on output in recessions.

In this paper we estimate a small rational expectations macroeconomic model of the U.S. economy in which monetary policy has temporary real effects due to sluggish adjustment in wages and prices. We then compare the stochastic properties of the economy in the presence of the zero bound on nominal interest rates when monetary policy is set according to an interest rate rule estimated over the 1990s but with alternative long-run inflation targets. We find that if the economy is subject to stochastic shocks similar in magnitude to those experienced in the U.S. over the 1980s and 1990s, the consequences of the zero bound constraint are negligible for target inflation rates as low as 2 percent. However, the effects of the constraint are non-linear with respect to the inflation target and become increasingly important for determining the effectiveness of policy with inflation targets between 0 and 1 percent. We find that economic performance deteriorates significantly with such low inflation targets. The variability of output increases noticeably, while the variability of inflation also rises somewhat. The stationary distribution of output is distorted with recessions becoming somewhat more frequent and longer lasting. Moreover, in our model the asymmetry of policy ineffectiveness induced by the zero bound generates a non-vertical long-run Phillips curve. Output falls increasingly short of potential, on average, as the inflation target, and therefore the average rate of inflation, becomes smaller. At zero average inflation, the output loss is in the order of 0.1 percent of potential output.

The remainder of this paper is organized as follows. Section 2 discusses interest rate rules and the role of money in the presence of the zero bound on nominal interest rates. Our estimated model of the U.S. economy is presented in section 3. Section 4 assesses

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the quantitative importance of the zero bound for stabilization policy based on stochastic simulation results. Section 5 concludes.

2 Monetary policy, money demand and the zero bound

Under normal circumstances, that is when the short-term nominal interest rate is not constrained at zero, monetary policy can be broadly characterized in terms of a Taylor-style interest rate rule. We have estimated a generalized form of such a policy rule for the United States over the 1980:Q1 to 1999:Q4 period, a period over which the zero bound has not constrained policy in any way (standard errors in parentheses):

$$i_t \;\; = \;\; -\,.0015 \;+\; .733\;i_{t-1} \;+\; .581\;\pi_t^{(4)} \;+\; 1.038\;y_t \;-\; .852\;y_{t-1} + \epsilon_{i,t}. \qquad (1)$$
$$ \underset{(.0028)}{} \quad \underset{(.062)}{} \qquad \underset{(.107)}{} \qquad \underset{(.239)}{} \qquad \underset{(.223)}{}$$

Here, i_t is the short-term interest rate, $\pi_t^{(4)}$ reflects the rate of change of the chain-weighted GDP deflator over four quarters ending in quarter t and y_t the output gap, based on the Congressional Budget Office (2002) estimate of potential output.

The estimated slope parameters in this policy rule capture the pattern of stabilization policy during the 1980s and 1990s. The estimated intercept (virtually zero) reflects the central bank's implicit inflation target, π^*, and equilibrium real interest rate, r^*, over this period. In particular, the policy rule may be rewritten as:

$$i_t \;\; = \;\; (1-.733)(r^*+\pi^*) + .733\,i_{t-1} + .581\,(\pi_t^{(4)}-\pi^*) + 1.038\,y_t - .852\,y_{t-1} + \epsilon_{i,t} \quad (1')$$

with the implicit relationship, $0 = (1-.733)(r^*+\pi^*) - .581\pi^*$, connecting these concepts. For example, the estimation suggests an implicit inflation target of 1.7 percent over this period, assuming a value of 2 percent for r^*.

In this description of policy, the money supply is hidden in the background. As long as the short-term interest rate is not constrained by the zero bound, the central bank can be viewed as providing liquidity as needed to achieve the desired interest rate prescribed by the interest rate rule (1). The appropriate quantity of the monetary base required for this can be determined from the relevant money demand equation. The details of that specification

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are not important for modeling policy if the monetary transmission channel can be described in terms of interest rates, as is usually the case in macroeconometric models used for policy analysis. To illustrate this point define the inverse of the GDP velocity of the monetary base (the Marshallian K), $K_t = M_t/P_t Q_t$, where M_t is the monetary base, Q_t is real GDP and P_t the GDP deflator, and consider the simple money demand relation (for the log of K_t, k_t):

$$i_t - i^* = -\kappa(k_t - k^*) + \epsilon_{k,t}.$$

Here $i^* = r^* + \pi^*$ and k^* denote the corresponding equilibrium levels that would obtain if the economy were to settle down to the policymaker's inflation target π^*, and $\epsilon_{k,t}$ summarizes other short-term influences to the demand for money.

Although this equation may usefully summarize the relation between money and interest rates when the short rate is above zero, once the zero bound is reached further injections of liquidity are no longer reflected in the short-term interest rate. Simply, market participants need not accept negative interest rates as currency can always serve as an alternative asset with a zero rate. A complete description of interest rates ought to reflect the zero bound constraint:

$$i_t = [\, i^* - \kappa(k_t - k^*) + \epsilon_{k,t}\,]_+, \tag{2}$$

where the function $[\,\cdot\,]_+$ imposes the bound. **Figure 1** illustrates the resulting non-linearity in the relationship between the monetary base and the short-term interest rate by drawing on the historical experience of the United States in the 1930s and in Japan in the 1990s. The figure shows annual data for the three-month interest rate and the Marshallian K of the monetary for each country. As can be seen, the usual downward relationship between the Marshallian K and the short-term interest rate evident under normal circumstances (in each case this reflects the early years of the sample) was distorted at the zero bound.

Similarly, characterizations of monetary policy with the interest rate rule (1) or ($1'$) must account for the zero bound in stochastic policy evaluation experiments. In particular, in an economy that is otherwise neutral to the policymaker's choice of a long-run inflation

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Figure 1: Monetary Base and the Zero Interest Rate Bound

United States: 1921-1940   Japan: 1981-2000
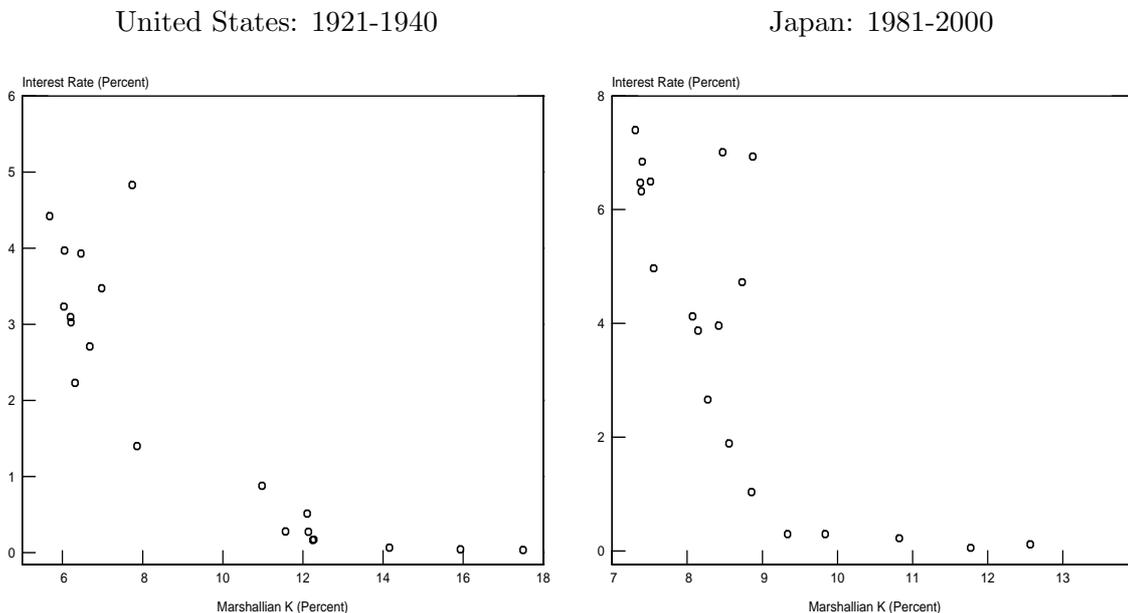
target, π^*, that choice influences the interest-rate easing buffer available for countercyclical policy $i^* = r^* + \pi^*$ and is therefore a determining factor of the extent to which the zero bound influences stabilization performance. Examining the performance of the economy for alternative values of π^* when policy follows a rule such as $(1')$, with an imposed zero bound, provides a benchmark for assessing the quantitative implications of the zero bound for stabilization policy. In what follows, we perform this exercise with a model designed to describe the U.S. economy over the 1980s and 1990s.

3   An empirical model of the U.S. economy

The small open economy model that we use as a laboratory for assessing the effectiveness of monetary policy when the nominal interest rate is constrained at zero incorporates forward-looking behavior by economic agents in labor markets, financial markets and goods

markets.[4] Expectations of endogenous variables are formed rationally and fully reflect the choice of monetary policy rule. Monetary policy, however, still has temporary real effects due to the presence of staggered wage contracts which induce nominal rigidity. The policy instrument (the nominal short-term interest rate) is set according to the estimated rule $(1')$ presented in the preceding section. Due to the presence of nominal rigidity, monetary policy affects the real interest rate and the real exchange rate, which in turn affect the various components of aggregate demand. Deviations of aggregate demand from potential output then have consequences for wage and price setting.

The model equations are summarized in **Table 1**. First, the long-term nominal rate, l_t, is related to expected future short-term rates via the term structure relationship in equation (3).[5] Then, the long-term real interest rate, r_t, is determined according to the Fisher equation (4), where p_t refers to the price level. The real exchange rate, s_t, depends on the differential between domestic and foreign real interest rates consistent with uncovered interest rate parity (5). The tilde ' ~ ' refers to foreign variables.

Aggregate demand is broken down into its major components: aggregate consumption, fixed investment, inventory investment, total (federal, state and local) government purchases and net exports, as indicated by equation (6). We scale each demand component by the level of potential output as estimated by the Congressional Budget Office (2002), and denote the result with lower-case letters. Normalized consumption, c_t, is modeled as a function of its own lagged value, permanent income and the expected long-term real interest rate in equation (7). The lagged dependent variable can be rationalized as reflecting habit persistence. Permanent income, $\bar{y}_t$, is modeled as the annuity value of expected income in the current and next eight periods. Fixed investment, f_t, depends on three lags of itself,

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[4]Earlier versions of this model where used in Orphanides, Small, Wieland and Wilcox (1997) as well as Levin, Wieland and Williams (1999, 2003). The model specification is broadly similar to Taylor (1993).

[5]Rather than estimating the term structure explicitly, we rely on the accumulated forecasts of the short rate over the following 8 quarters which, under the expectations hypothesis, will coincide with the long rate forecast for this horizon. In defining the long rate in terms of the expectations hypothesis we deliberately avoid the added complexities that would be associated with modeling term and risk premia. Since our specification is invariant to the presence of a constant premium, we set it equal to zero for expositional simplicity.

<div align="center">Table 1: Model Equations</div>

Interest and Exchange Rates

Long-Term Nominal Rate

$$l_t = \mathrm{E}_t \left[\tfrac{1}{8} \sum_{j=1}^{8} i_{t+j-1} \right] \tag{3}$$

Long-Term Real Rate

$$r_t = l_t - 4\,\mathrm{E}_t \left[\tfrac{1}{8} \left(p_{t+8} - p_t \right) \right] \tag{4}$$

Real Exchange Rate

$$s_t = \mathrm{E}_t\left[s_{t+1} \right] + 0.25 \left(i_t - 4\,\mathrm{E}_t\left[p_{t+1} - p_t \right] \right)$$
$$- 0.25 \left(\tilde{i}_t - 4\,\mathrm{E}_t\left[\tilde{p}_{t+1} - \tilde{p}_t \right] \right) \tag{5}$$

Aggregate Demand Components

Aggregate Demand

$$y_t = c_t + f_t + n_t + e_t + g_t - 1 \tag{6}$$

Consumption

$$c_t = \alpha_1\, c_{t-1} + \alpha_2\, \bar{y}_t + \alpha_3\, r_t + \epsilon_{c,t}, \tag{7}$$

$$\text{where } \bar{y}_t = \frac{(1-.9)}{1-(.9)^9} \sum_{i=0}^{8} (.9)^i y_{t+i}$$

Fixed Investment

$$f_t = \sum_{i=1}^{2} \beta_i\, f_{t-i} + \beta_3\, \bar{y}_t + \beta_4\, r_t + \epsilon_{f,t} \tag{8}$$

Inventory Investment

$$n_t = \sum_{i=1}^{3} \gamma_i\, n_{t-i} + \sum_{i=1}^{3} \gamma_{3+i}\, y_{t-i-1} + \epsilon_{n,t} \tag{9}$$

Net Exports

$$e_t = \delta_1 e_{t-1} + \delta_2 y_t + \delta_3 y_t^* + \delta_4 s_t + \epsilon_{e,t} \tag{10}$$

Government Spending

$$g_t = \rho g_{t-1} + \epsilon_{g,t} \tag{11}$$

Prices and Wages

Price Level

$$p_t = \sum_{i=0}^{3} \omega_i\, x_{t-i}, \tag{12}$$

$$\text{where } \omega_i \geq 0,\ \omega_i \geq \omega_{i+1} \text{ and } \sum_{i=0}^{3} \omega_i = 1$$

Contract Wage

$$x_t = \mathrm{E}_t \left[\sum_{i=0}^{3} \omega_i\, v_{t+i} + \chi \sum_{i=0}^{3} \omega_i\, y_{t+i} \right] + \epsilon_{x,t}, \tag{13}$$

$$\text{where } v_t = \sum_{i=0}^{3} \omega_i \left(x_{t-i} - p_{t-i} \right)$$

Notes: l: long-term nominal interest rate; i: short-term nominal interest rate; r: ex-ante long-term real interest rate; p: aggregate price level; s: real exchange rate; y: output gap; c: consumption; $\bar{y}$: permanent income; f: fixed investment; n: inventory investment; e: net exports; g: government spending; x: nominal contract wage; v: real contract wage index; $\epsilon_{(.)}$: random white-noise shocks; the tilde ' ˜ ' indicates foreign variables.

permanent income as a measure of expected future sales, and the real interest rate (equation (8)), while inventory investment, n_t, instead is (nearly) of the accelerator type (equation (9)). Net exports, e_t, depend on the level of income at home and abroad, and on the (trade-weighted) real exchange rate (equation (10)). Finally, government spending, g_t, follows a simple autoregressive process with a near-unit root (equation (11)). (Random white noise shocks are denoted by $\epsilon_{.,t}$).

As to the short-run supply-side of the model we follow Fuhrer and Moore (1995a,b) rather than Taylor (1980) in modeling staggered wages and prices. Fuhrer and Moore assume that workers and firms set the real wage in the first period of each new contract with an eye toward the real wage agreed upon in contracts signed in the recent past and expected to be signed in the near future.[6] As they show, models specified in this manner exhibit a greater (and hence more realistic) degree of inflation persistence than do models in which workers and firms care about relative wages in nominal terms. Equation (12) indicates that the price level is related to the weighted average of wages on contracts that are currently in effect assuming a constant markup. Equation (13) specifies that the real wage under contracts signed in the current period, $x_t - p_t$, is set in reference to a centered moving average of initial-period real wages established under contracts signed as many as three quarters earlier as well as contracts to be signed as many as three quarters ahead. Furthermore, the negotiated real wage is assumed to depend also on expected excess-demand conditions. The maximum contract length is four quarters.

In the deterministic steady state of this model output is at potential and the sectoral allocation of GDP is constant for a given combination of equilibrium real interest and exchange rates. The steady-state value of inflation is determined exclusively by the inflation target and the policy rule, because the wage-price block does not impose any restriction on the steady-state inflation rate.

Model estimation. The model allows for inflation and output persistence. While the

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[6]By contrast, Taylor assumed that workers and firms set the *nominal* wage in the first period of each new contract with an eye toward the *nominal* wage settlements of recently signed and soon-to-be signed contracts.

presence of these lags is not explicitly derived from optimizing behavior of representative agents they are consistent with the presence of habit persistence in consumption, adjustment costs in investment and overlapping wage contracts. The advantage of such a model is that it can fit empirical inflation and output dynamics for the U.S. economy up to a set of white-noise structural shocks.[7] The demand side equations are estimated on an equation-by-equation basis using instrumental variables. As to the supply side, we follow Fuhrer and Moore (1995a,b) and use price data in estimation. We estimate the parameters of the wage-price block by simulation-based indirect inference methods so as to fit the empirical output and inflation dynamics as summarized by an atheoretical VAR model.[8] The individual equations fit the data well. In addition we have evaluated the overall fit of the complete model. The series of historical structural shocks computed under model-consistent expectations show no remaining serial correlation. Furthermore, the degree of inflation and output persistence implied by the model fits the observed degree of persistence as summarized by an unconstrained VAR model. Individual parameter estimates and evidence regarding the empirical fit of the model are presented in the appendix.

Global stability and fiscal policy. The zero bound constraint is the only effective nonlinearity in the model. However, when it is introduced, the global stability of our otherwise linear system is no longer ensured. Once shocks to aggregate demand or supply push the economy into a sufficiently deep deflation, a zero-interest-rate policy may not be able to return the economy to the original equilibrium. With a series of shocks large enough to sustain deflationary expectations and to keep the real interest rate above its equilibrium level, aggregate demand is suppressed further sending the economy into a deflationary spiral. This points to a limitation inherent in linear models such as this which rely on the real

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[7]An alternative approach following Rotemberg and Woodford (1997) is to estimate a model with optimizing agents and achieving empirical fit by introducing ad-hoc serially correlated shocks as criticized by Estrella and Fuhrer (2000). In both cases, the degree of output and inflation persistence is important for the analysis of monetary policy.

[8]For a more detailed discussion of this estimation methodology see Coenen and Wieland (2000). We investigated both Taylor's (1980) as well as Fuhrer and Moore's (1995) specification. Our findings confirmed the earlier results of Fuhrer and Moore, who showed that under the assumption of rational expectations and perfect credibility of monetary policy Taylor's specification does not induce sufficient inflation persistence to match U.S. data.

interest rate as the sole channel for monetary policy and also brings into focus the extreme limiting argument regarding the ineffectiveness of monetary policy in a liquidity trap.

To ensure global stability in the presence of the zero-bound constraint, we introduce a second nonlinearity. We specify a fiscal policy that, if deflation becomes so severe that the zero bound restricts the real interest rate at a level high enough to induce a growing aggregate demand imbalance, boosts aggregate demand to rescue the economy from falling into a deflationary spiral.[9]

4 The quantitative importance of the zero bound

To evaluate whether the zero bound on nominal interest rates would be of quantitative significance in practice, it is necessary to assess how frequently monetary policy would be expected to be constrained if the economy were subjected to stochastic shocks with properties similar to those we anticipate to obtain in practice. To this end, we employ stochastic simulations of our model economy. As a baseline, we assume the economy is subject to shocks drawn from a joint normal distribution with the covariance of the shocks we estimated for the 1980s and 1990s.[10] With these simulations we construct the stationary distribution of interest rates, inflation and output and investigate the extent to which their statistical properties are altered when the policymaker adopts alternative values of the inflation target, π^*, in the estimated policy rule (1′). In particular, we examine the influence of the inflation target on the means and variances of inflation and output, which would be central for welfare analysis based on a quadratic loss function. The equilibrium real interest rate r^* will be maintained at 2 percent.[11]

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[9]The extent of fiscal impetus is related to the deviation of the actual federal funds rate, i_t, (which cannot be negative), from the notional rate, i_t^n, that would be prescribed by the estimated interest rate rule in the absence of the zero bound. The fiscal impetus comes into play with a half-year delay and responds only to a moving average of of negative deviations of the prescribed interest rate from zero. To ensure fiscal consolidation in the long-run, we also restrain government expenditure in a symmetric fashion whenever the economy experiences very favorable economic conditions, that is, in a situation when output is so far above potential that the interest rate rule prescribes a rate of more than twice the steady-state value.

[10]The derivation of historical shocks and the solution methodology are discussed in the appendix.

[11]Nevertheless, it is straightforward to assess the effect of alternative values of r^*. The zero bound regards the nominal interest rate which in deterministic steady state equals the sum of r^* and π^*. Thus, changes in one parameter can be offset by changes in the other. For example, our results for π^* equal to 1 percent with

Figure 2: Distortion of Stationary Distribution of Nominal Interest Rate
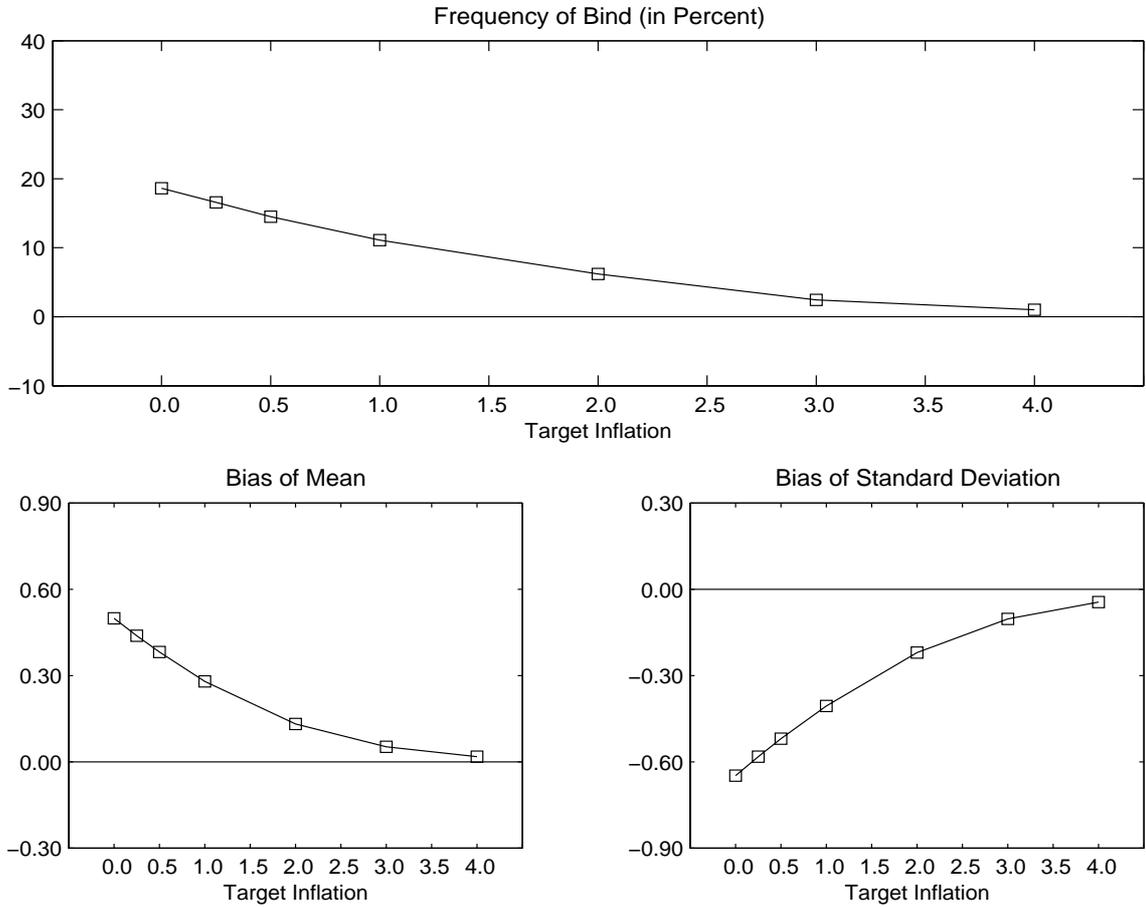
Figure 2 shows the impact of the inflation target on the distribution of the nominal interest rate. The top panel shows the frequency with which the zero bound constrains monetary policy, that is the frequency with which the monetary authority would have set the nominal rate below zero if that were feasible in that period. As can be seen, the zero bound does not represent a quantitatively important factor at inflation targets at or above two percent. The constraint becomes binding with about one-tenth frequency only for targets close to or below one percent. However, this frequency increases to 20 percent as

our baseline assumption of r^* equal to 2 percent also describe the outcome in an economy with r^* equal to 1 percent and π^* equal to 2 percent.

the inflation target drops towards zero.

The bottom panels of **Figure 2** describe the resulting distortion of the stationary distributions of the nominal interest rate. The bottom left panel shows the distortion in the average level of the nominal interest rate. This is computed as the mean of the stationary distribution of the short nominal interest rate, i_t, minus $r^* + \pi^*$, which corresponds to the mean nominal rate in the absence of the constraint. This property is indeed confirmed in the figure with the constraint in place when the inflation target is large enough for the bind to occur very infrequently. With inflation targets near zero, however, the asymmetric nature of the constraint on policy introduces a significant bias. Since the constraint provides a lower bound on the nominal interest rate, it effectively forces policy to be tighter than it would be in the absence of the constraint under some circumstances. Since no comparable upper bound is in place, policy is tighter on average. This bias increases with the frequency with which the constraint binds. As can be seen from the figure, a policymaker following the estimated rule with a zero inflation target would set the nominal interest rate about 50 basis points higher, on average, than if the zero-bound constraint were not in place. Furthermore, since this constraint restricts the variability of interest rates, the standard deviation of the interest rate falls somewhat as the inflation target drops to zero as shown in the bottom-right panel of **Figure 2**.

The distortions of the distribution of nominal interest rates translate into distortions of the stationary distributions of output and inflation. Compared to the unconstrained case, in which the distributions would be normal, the left tails of the output and inflation distributions will be noticeably thicker. When either output or inflation fall considerably below their means, policy without the constraint would engineer an easing in order to return output to potential and inflation to its target level. With the constraint binding, this is no longer feasible and consequently reflation of the economy occurs at a slower pace. Summary information regarding the distortion of the distributions of output and inflation with the inflation target is shown in **Figure 3.** The top panel shows the resulting bias in the means of output and inflation and the bottom panel the corresponding changes in the standard

Figure 3: Distortion of Stationary Distributions of Output and Annual Inflation

Output

Annual Inflation
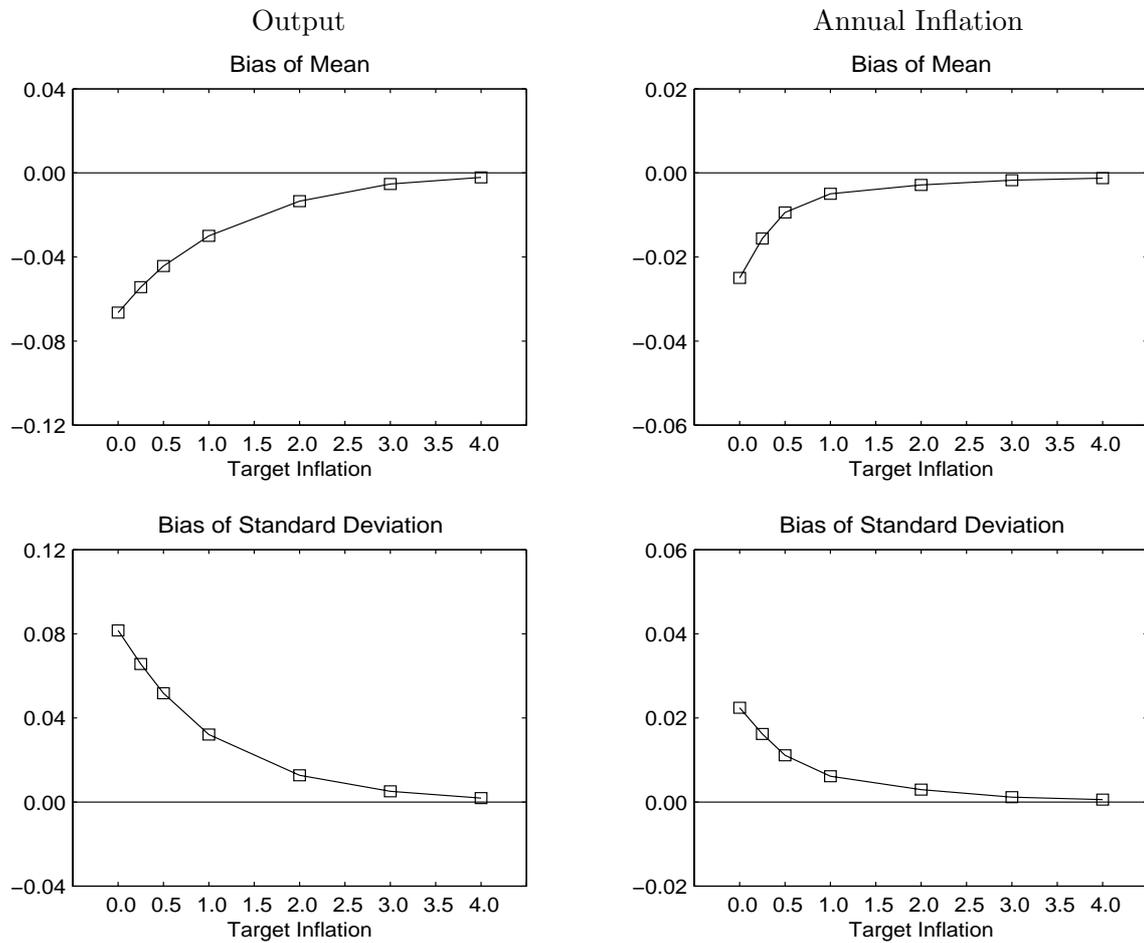
deviations. As shown in the upper-right panel, a small downward bias in average inflation (relative to the target) appears as a result of the zero bound. Such a bias is not materially significant, however, since a small adjustment to the inflation target in the policy rule could yield any desired average level of inflation. A more significant bias materializes with respect to the output gap. As the inflation target drops to zero, output fails to reach potential, on average, resulting in a negative average output gap. For a zero inflation target the average output loss is a little below one tenth of a percent. As the bottom panels of the figure suggest, the variability of output and inflation also increases at near zero inflation targets.

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Figure 4: Implicit Long-Run Phillips Curve
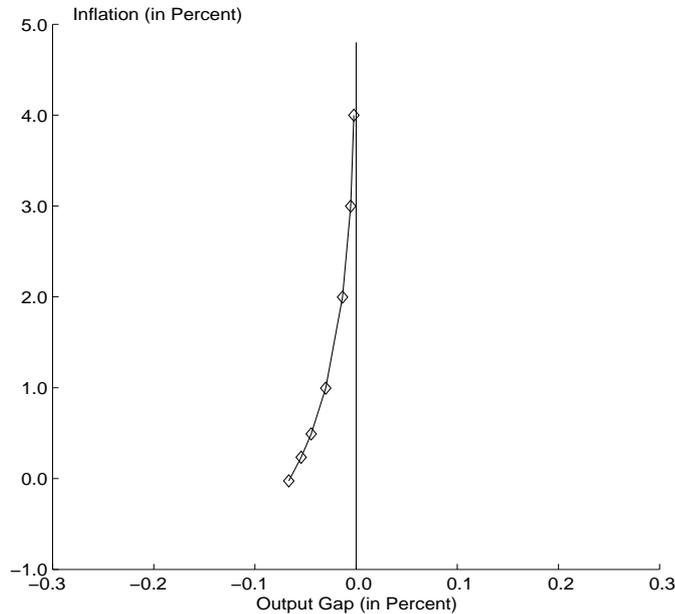
The presence of the zero-bound constraint in our model clearly invalidates the long-run superneutrality that obtains in a linear version of the model. The relationship between the average level of output and the average level of inflation that is due to the zero bound implies the existence of a long-run Phillips curve. This is shown in **Figure 4** which plots the upward sloping relationship between average inflation and average output.[12] To note, the slope of the long run Phillips curve generated by the zero-bound constraint is only noticeable at average inflation rates below two percent and is fairly small. More important, perhaps, is the non-linearity in the schedule suggesting a greater loss at the margin for additional reductions in the inflation target as the inflation target and average inflation fall towards zero.

The source of this non-neutrality can be directly traced to the interaction between the policy rule and the forward-looking nature of expectations in our model. As is well

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[12]Employing Okun's law to translate negative output gaps to positive unemployment gaps would generate a downward sloping long-run Phillips curve in the more traditional inflation-unemployment space.

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known, in models with rational expectations such as ours, the sacrifice ratio—the ratio of the cumulative output gap loss (gain) required for a given reduction (rise) in the inflation rate—is a function of the policy responsiveness to inflation and output. With a linear policy reaction function, as is the case when the inflation target is sufficiently high for the zero bound to be irrelevant, output losses when inflation is above the steady state and falls towards it exactly offset output gains when inflation is below the steady state and rising towards it. The responsiveness of policy to inflation and output is the same in both cases. Symmetry prevails and on average the output gap is zero. This is not the case when the zero bound becomes important. When the constraint is binding, the responsiveness of policy to marginal changes in inflation is nil—the interest rate is constrained at zero. When the constraint is not binding, the usual responsiveness of policy is restored. But the former is more likely when inflation is below its target than above its target so symmetry fails and a bias in the average output gap appears. It is worth noting that if expectations were of a backward-looking, adaptive nature, the long-run Phillips curve would be vertical as in that case the sacrifice ratio would be invariant to the policy responsiveness altogether. Of course, introducing additional non-linearities in policy might offset this bias but it would also move the policy away from its original unrestricted linear specification and distort the higher moments of the stationary distributions of inflation and output.

5   Conclusion

Our analysis for the United States indicates that if the economy is subject to stochastic shocks similar in magnitude to those experienced over the 1980s and 1990s, the consequences of the zero bound are negligible for target inflation rates as low as 2 percent. However, the effects of the constraint become increasingly important for determining the effectiveness of policy with inflation targets between 0 and 1 percent. Although these results are suggestive, it is important to recognize that some uncertainty remains regarding the magnitude of the distortions introduced by the zero bound when targeting zero inflation. For example, since our model was estimated for the 1980s and 1990s, a relatively calm period for the U.S.

economy, the variances of demand and supply shocks may be smaller than in earlier periods. Larger disturbances will render the zero bound more important. Similarly, the assumption that policymakers observe the data and the relevant model parameters without error may lead us to underestimate the impact of the zero bound. Recognition of data uncertainty (see for example Orphanides (2001) or parameter uncertainty (see for example Wieland (1998)) would raise the importance of the zero bound as a constraint on monetary policy in practice. However, our estimate would be reduced to the extent that channels of monetary policy transmission other than the interest or exchange rate channel would remain effective important when the zero bound renders the interest rate channel ineffectual. Similarly, policy outcomes might be improved if a non-linear policy rule for the interest rate or for the exchange rate designed to explicitly reduce the distortions resulting from the zero bound were followed.

In summary, our results point to a fundamental difficulty associated with the evaluation of stabilization policies with a price stability objective based on simple linear models. The presence of the zero bound constraint invalidates the underlying superneutrality properties of otherwise linear models. At low rates of inflation, the zero bound distorts the stochastic properties of the economy and induces a tradeoff between the average level of inflation and the variability of inflation and output. As a result, the optimal average rate of inflation cannot be investigated independently of the variability of output and inflation. Since our results suggest that deflation potentially engenders greater dangers than inflation, it may be optimal to pursue a price stability objective that allows for a small but positive bias in the average rate of inflation. The optimal size of such a bias, however, remains an open question. Furthermore, the optimal policy rule in the presence of the zero bound on nominal interest rates is likely to be nonlinear and asymmetric in a low or zero inflation environment. Characterizing the optimal rule represents an important issue for future research.

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Appendix: Estimation results and simulation techniques

The parameter estimates of our model are summarized in **Table A-1**.

Table A-1: Parameter Estimates

Consumption [a]	α_1	α_2	α_3			
	0.642	0.304	-0.062			
	(0.045)	(0.038)	(0.015)			
Fixed Investment [a]	β_1	β_2	β_3	β_4		
	1.383	-0.408	0.030	-0.019		
	(0.049)	(0.053)	(0.015)	(0.014)		
Inventory Investment [a]	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
	0.350	0.072	0.129	0.315	-0.084	-0.188
	(0.058)	(0.041)	(0.078)	(0.050)	(0.073)	(0.035)
Net Exports [a]	δ_1	δ_2	δ_3	δ_4		
	0.907	-0.027	0.056	-0.006		
	(0.040)	(0.019)	(0.011)	(0.002)		
Government Spending [a]	ρ					
	0.956					
	(0.024)					
Fuhrer-Moore Contracts [b]	ω_0	ω_1	ω_2	ω_3	χ	
	0.451	0.302	0.1237	0.1237	0.003	
	(0.054)	(0.016)			(0.0015)	

Notes: [a] Instrumental variables estimates. Standard errors in parentheses. Sample period: 1980:Q1 to 1999:Q4. [b] Simulation-based indirect estimates using a VAR(3) model of quarterly inflation and the output gap as auxiliary model. Standard errors in parentheses. Sample period: 1965:Q1 to 2001:Q4 extending Fuhrer and Moore (1995).

In preparation for the stochastic simulations, we first computed the structural shocks of the model based on U.S. data from 1980 to 1999.[13] Since the non-negativity constraint for nominal interest rates was never binding during this period and our model is otherwise linear, we obtained the structural shocks by solving the model analytically for the reduced form using the AIM implementation (Anderson and Moore, 1985, and Anderson, 1997) of the Blanchard and Kahn (1980) method for solving linear rational expectations models. The structural shocks also provide a good indication of the historical fit of our model. **Figure A-1** shows the correlogram of historical structural shocks, which overall reveals no significant serial correlation.

A further indication of the good empirical fit of our model is obtained from a comparison of the implied autocorrelation functions of inflation and output with the empirical autocorrelation functions implied by an unconstrained bivariate VAR.[14] The comparison of autocorrelation functions of inflation and output in the U.S. economy is reported in **Figure A-2**. The solid lines refer to the autocorrelation functions implied by the model. The thin dotted lines in each panel correspond to the asymptotic 95% confidence bands associated with the autocorrelation functions of the bivariate unconstrained VAR(3) model used in the estimation of the staggered contracts specifications.[15]

Based on the covariance matrix of structural historical shocks, we generated 100 sets of artificial normally-distributed shocks with 100 quarters of shocks in each set from which the first 20 twenty quarters of shocks were discarded in order to guarantee that the effect of the initial values die out. We then used the sets of retained shocks to conduct stochastic simulations under alternative inflation targets, while imposing the non-negativity constraint on nominal interest rates.[16]

We simulate the model using an efficient algorithm implemented in TROLL and based on work by Boucekkine (1995), Juillard (1994) and Laffargue (1990). It is related to the Fair-Taylor (1983) extended path algorithm. A limitation of the algorithm is that the model-consistent expectations of market participants are computed under the counterfactual

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[13]The process of calculating the structural shocks would be straightforward if the model in question were a purely backward-looking model. For a rational expectations model, however, structural shocks can be computed only by simulating the full model and computing the time series of model-consistent expectations with respect to historical data. The structural shocks differ from the estimated residuals to the extent of agents' forecast errors.

[14]Such an approach has also been used by Fuhrer and Moore (1995a) who argued that autocorrelation functions are more appropriate for confronting macroeconomic models with the data than impulse response functions because of their purely descriptive nature.

[15]For a detailed discussion of the methodology and the derivation of the asymptotic confidence bands for the estimated autocorrelation functions the reader is referred to Coenen (2000).

[16]If it were not for this nonlinearity, we could use the reduced form of the model corresponding to the alternative policy rules to compute unconditional moments of the endogenous variables without having to resort to stochastic simulations.
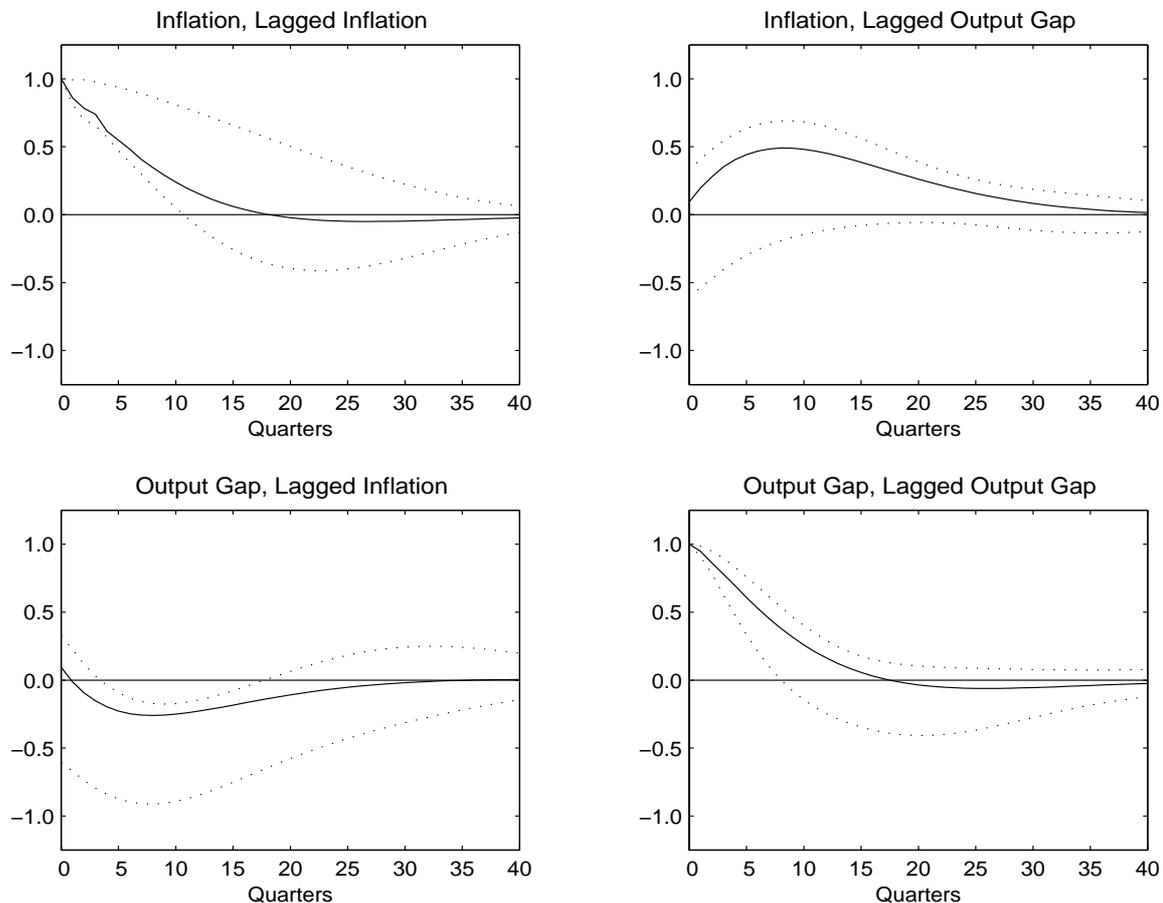
Figure A-1: Correlation Pattern of Historical Structural Shocks
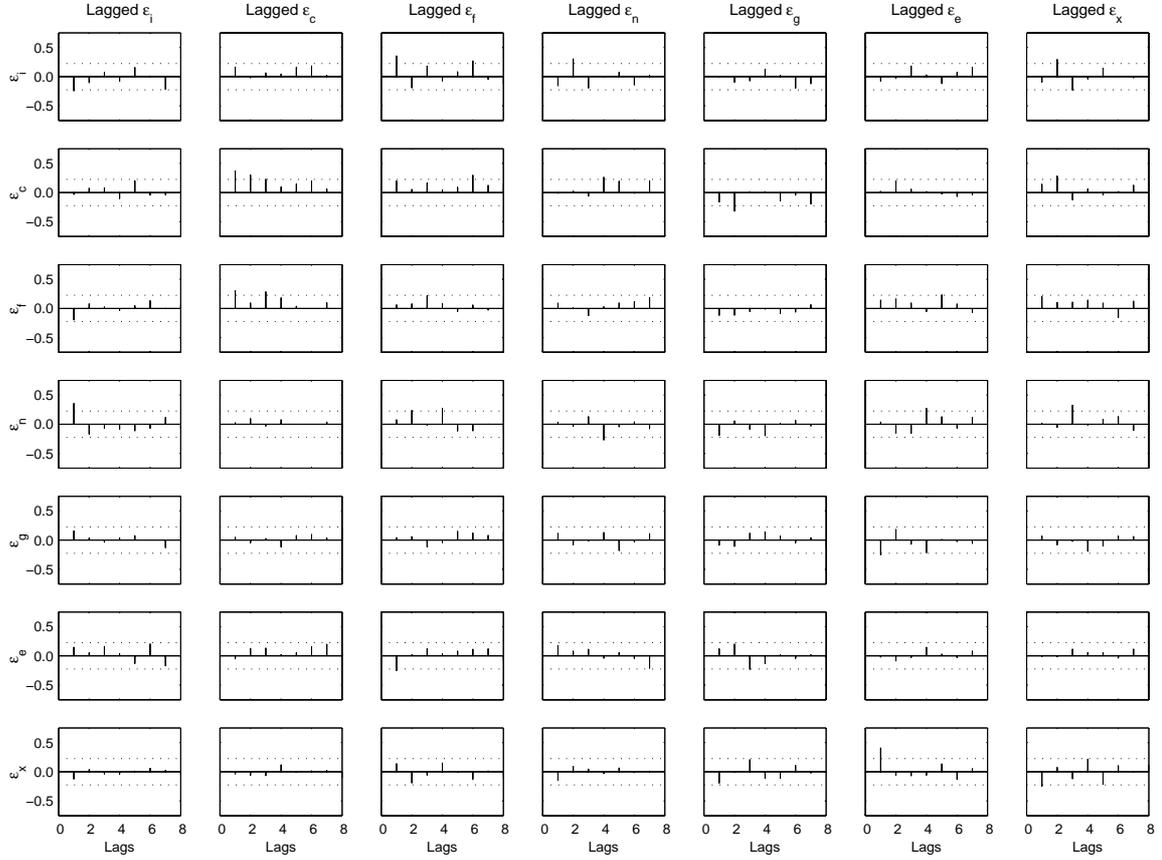
Notes: Solid bars: Autocorrelation functions implied by the estimated model of the U.S. economy. Dotted lines: Asymptotic 95%-confidence bands.

assumption that 'certainty equivalence' holds in the nonlinear model being simulated. This means, when solving for the dynamic path of the endogenous variables from a given period onwards, the algorithm sets future shocks equal to their expected value of zero. Thus the variance of future shocks has no bearing on the formation of current expectations and economic performance. This would be correct in a linear model. However once we introduce the zero bound on nominal interest rates into the model, we are able to show that the variance of future shocks ought to be expected to introduce a small bias in the average levels of various variables, including importantly, interest rates. This result is discussed

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Figure A-2: Fitting Inflation and Output Dynamics with the Structural Model

in detail in section 4 of this paper. To be clear, we should emphasize that the variance
of shocks has both a direct and an indirect effect on the results. The direct effect is that
a greater variance of shocks implies that the zero bound on nominal interest rates binds
with greater frequency, the indirect effect is that all agents should be taking this effect of
the variance into account when they form their expectations. The simulation algorithm
captures the direct effect but not the indirect one.

There are other solution algorithms for nonlinear rational expectations models that do

not impose certainty equivalence. But these alternative algorithms would be prohibitively costly to use with our model, which has more than twenty state variables. Even with the algorithm we are using, stochastic analysis of nonlinear rational expectations models with a moderate number of state variables remains fairly costly in terms of computational effort.