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ROBUSTNESS OF SIMPLE MONETARY POLICY RULES UNDER MODEL UNCERTAINTY

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ABSTRACT

In this paper, we investigate the properties of alternative monetary policy rules using four structural macroeconometric models: the Fuhrer-Moore model, Taylor's Multi-Country Model, the MSR model of Orphanides and Wieland, and the FRB staff model. All four models incorporate the assumptions of rational expectations, short-run nominal inertia, and long-run monetary neutrality, but differ in many other respects (e.g., the dynamics of prices and real expenditures). We compute the output-inflation volatility frontier of each model for alternative specifications of the interest rate rule, subject to an upper bound on nominal interest rate volatility. Our analysis provides strong support for rules in which the first-difference of the federal funds rate responds to the current output gap and the deviation of the one-year average inflation rate from a specified target. In all four models, first-difference rules perform much better than rules of the type proposed by Taylor (1993) and Henderson and McKibbin (1993), in which the level of the federal funds rate responds to the output gap and inflation deviation from target. Furthermore, first-difference rules generate essentially the same policy frontier as more complicated rules (i.e., rules that respond to a larger number of variables and/or additional lags of output and inflation). Finally, this class of rules is robust to model uncertainty, in the sense that a first-difference rule taken from the policy frontier of one model is very close to the policy frontier of each of the other three models. In contrast, more complicated rules are less robust to model uncertainty: rules with additional parameters can be fine-tuned to the dynamics of a specific model, but typically perform poorly in the other models.

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1. Introduction

In the face of uncertainty about the true structure of the economy, policymakers may disagree about the macroeconomic effects of monetary policy and thus about the appropriate policy setting. One approach to resolving this problem is to search for monetary policy rules that work well across a wide range of structural models; that is, rules that are robust to model uncertainty.[1] In this paper, we investigate the characteristics of policy rules that yield low output and inflation volatility across four different structural macroeconometric models of the U.S. economy: the FRB staff model (cf. Brayton 1997a), the MSR model of Orphanides and Wieland (1997),[2] the Fuhrer-Moore (1995) model (henceforth referred to as the FM model), and Taylor's (1993b) Multi-Country Model (henceforth TAYMCM). All four models incorporate the assumptions of rational expectations, short-run nominal inertia, and long-run monetary neutrality, but differ in many other respects (e.g., the dynamics of prices and real expenditures). We compute the inflation-output volatility frontier of each model for alternative specifications of the interest rate rule, subject to an upper bound on nominal interest rate volatility. We then evaluate robustness to model uncertainty by taking the rules that perform well in one model and measuring their performance in each of the other three models.

Our analysis provides strong support for rules in which the first-difference of the federal funds rate responds to the current output gap and the deviation of the one-year average inflation rate from a specified target. First, in all four models, first-difference rules perform much better than rules of the type considered by Taylor (1993) and Henderson and McKibbin (1993), in which the level of the federal funds rate responds to the output gap and inflation deviation from target. Second, more complicated rules (that is, rules that respond to a larger number of variables and/or additional lags of the output gap and inflation) typically generate very small gains in stabilizing output and inflation compared with optimal first-difference rules. A closely related result is that rules involving model-based forecasts generally do not outperform first-difference rules based on the current output gap and inflation rate, and quite often generate higher variability of output and inflation. Finally, the class of first-difference rules is robust to model uncertainty, in the sense that a first-difference rule taken from the policy frontier of one model is very close to the policy frontier of each of the other three models. In contrast, we find that more complicated rules are somewhat less robust to model uncertainty: rules with a larger number of free parameters can be fine-tuned to the dynamics of a specific model, but often perform poorly in other models compared with the best simple rules.

The approach of evaluating policy rules used in this paper follows the long and distinguished tradition dating to Phillips (1954).[3] As is standard in this literature, we assume the objective of policy is to minimize the weighted sum of the unconditional variances of the inflation rate and the output gap (the percent deviation of GDP from its potential level). In addition, we allow that interest rate volatility may enter into the policymakers' optimization problem, either through preferences or constraints on policy actions. The funds rate is set according to a time-invariant policy rule. For a given class of policy rules, the policy frontier traces out the best obtainable outcomes in terms of inflation, output, and funds rate volatility. We refer to the policy rules underlying such a frontier as "optimal" in the sense that these rules represent solutions to the specified constrained optimization problem.

One major difference between our analysis and much of that in the previous literature is that we compute optimal policy frontiers using large rational expectations macroeconomic models--including models with more than 100 equations--as opposed to traditional structural models or small rational expectations models.[4] Policy rule analysis using traditional models is particularly prone to the Lucas Critique (1976). Fischer (1977) and Phelps and Taylor (1977) made strides in overcoming the inconsistency between policy and expectations inherent in traditional models by using small rational expectations structural models for policy analysis.[5] In the past, policy rule analysis using rational expectations models was hampered by the computational cost in solving and computing moments of models with more than a small number of equations. Analysis was generally limited to the comparison of a small set of policy regimes as in Bryant (1989), Bryant (1993), and Taylor (1993b). Increases in computer speed and the development of efficient solution algorithms have made the computation of optimal frontiers of large linear rational expectations models feasible.

We present the policy frontiers in inflation-output volatility space, with each curve corresponding to a particular constraint on the volatility of the first-difference of the funds rate. Interest rate volatility plays a key role in our analysis. All four models share the feature of a tradeoff between interest rate volatility and inflation-output volatility, even at levels of interest rate volatility significantly above those implied by estimated policy rules or observed in the data. That is, the variability of output and inflation can be reduced by using highly aggressive rules, but such rules also induce wild fluctuations in interest rates. In this paper, we focus our attention on rules that feature relatively moderate levels of interest rate volatility.

One argument for doing so is that the relatively low level of funds rate volatility seen in the data may be a consequence of a preference on the part of policymakers for low interest rate volatility. Even if no fundamental preference for low interest rate volatility exists, two reasons remain to focus on rules that generate moderate levels of interest rate volatility. First, linear policy rules that generate highly volatile interest rates proscribe frequent and large violations of the non-negativity constraint on the federal funds rate. In principle, one could analyze non-linear rules that incorporate this lower bound on interest rates, but doing so would raise the computational costs of our analysis by orders of magnitude.[6] Second, the hypothesized invariance of the estimated model parameters to changes in policy rules is unlikely to hold true under policies that are so dramatically different (in terms of funds rate volatility) from those seen during the sample periods over which the models were estimated.

The outline of the paper is as follows. Section 2 provides a brief description of the four models. Section 3 analyzes the inflation-output volatility frontier of each model for the following classes of policy rules: 3-parameter rules in which the funds rate responds to the current output gap, a moving average of the inflation rate, and the lagged funds rate; more complicated rules that incorporate a larger number of observed state variables; and rules that incorporate model-based forecasts of the output gap and inflation rate. This section also considers the extent to which these results are sensitive to the information lags that policymakers typically face. Section 4 analyzes the performance of other simple rules and investigates several potential explanations for the superior performance of rules with a coefficient near unity on the lagged interest rate. Section 5 compares the extent to which simple and complicated rules are robust to model uncertainty. Conclusions then follow.

2. Comparison of Basic Model Properties

Table 1 outlines the basic features of the four models with respect to the level of aggregation of the IS, price, labor, and external sector blocks, specification of price dynamics, the characteristics of the forward-looking terms in the IS block (bond duration and discounting for permanent income), and the estimation period. The behavioral equations of FM were estimated using FIML and a combination of OLS, 2SLS, and GMM were used for the other three models.

Aggregate Demand. FM represents aggregate spending by a single reduced-form equation corresponding to an IS curve. The current output gap depends on its lagged values over the past two quarters and the lagged value of the long-term real interest rate, which is defined as a weighted average of *ex ante* short-term real interest rates with duration equivalent to a 30 year coupon bond, with estimated coefficients taken from Fuhrer (1997a). FM does not explicitly include trade variables or exchange rates; instead, net exports (and the relationship between real interest and real exchange rates) are implicitly incorporated in the IS curve equation.

The MSR model disaggregates real spending into five components: private consumption, fixed investment, inventory investment, net exports, and government purchases.[7] The IS components exhibit partial adjustment to their respective equilibrium levels, measured as shares of potential GDP. Equilibrium consumption is a function of permanent income (discounted 10% per quarter) and the real (two-year) bond rate, equilibrium fixed investment is a function of output growth and the real bond rate, and equilibrium inventory investment depends only on output growth. Equilibrium government purchases are a constant share of GDP. Net exports are assumed to be fixed in the simulations reported here.

The TAYMCM model disaggregates IS components further; for example, spending on fixed investment is separated into three components: equipment, nonresidential structures, and residential construction. The specification of these equations is very similar to that of the more aggregated equations in the MSR model. In TAYMCM, imports follow partial adjustment to a equilibrium level that depends on U.S. income and the relative price of imports, while exports display partial adjustment to an equilibrium level that depends on foreign output and the relative price of exports. Uncovered interest rate parity determines each bilateral exchange rate (up to a time-varying risk premium); e.g., the expected one-period-ahead percent change in the DM/$ exchange rate equals the current difference between U.S. and German short-term interest rates.

The FRB model features about the same level of aggregation as TAYMCM for private spending but divides government spending into six components, each of which follows a simple reduced-form equation that includes a counter-cyclical term. The specification of most non-trade private spending equations follows Tinsley's (1993) generalized adjustment cost model. Each component has a specific flow or stock equilibrium condition; for example, equilibrium aggregate consumption is proportional to wealth.[8] Households and businesses adjust their spending in each category according to the solution of a quadratic adjustment cost problem. The resulting spending decision rules take a forward-looking error correction form where current spending growth depends on its own lags (up to three), expected future growth in equilibrium spending, and (negatively) on the log difference between lagged spending and its equilibrium level. Exports and non-oil imports are specified as error-correction processes where equilibrium real exports are proportional to the ratio of foreign GDP to the real exchange rate, and equilibrium real imports are proportional to the product of domestic GDP and the real exchange rate. Uncovered interest rate parity determines the multilateral exchange rate up to a sovereign risk premium that moves with the net foreign asset position of the United States.

Aggregate Supply. In FM, MSR, and TAYMCM, the aggregate wage rate is determined by overlapping wage contracts. In particular, the aggregate wage is defined to be the weighted average of current and three lagged values of the contract wage rate. TAYMCM follows the specification in Taylor (1980), where the current nominal contract wage is determined as a weighted average of expected nominal contract wages, adjusted for the expected state of the economy, in effect during the life of the contract. FM and MSR use the specification due to Fuhrer and Moore (1995), where the real contract wage--the contract wage deflated by the aggregate wage--is determined as a weighted average of expected real contract wages, adjusted for the expected level of the output gap, in effect over the life of the contract.[9]

In FM and MSR, the aggregate price level is a constant markup over the aggregate wage rate. In contrast, the output price in TAYMCM takes the (backward-looking) error-correction formulation where current price inflation depends positively on its own lag, current wage inflation, and lagged import price inflation and negatively (with a relatively large coefficient of -0.2) on the lagged deviation of the log of the price level from its equilibrium value. Import prices error-correct slowly to an equilibrium level equal to a constant markup over a weighted average of foreign prices converted to dollars. This partial adjustment of import and output prices imparts slightly more persistence to output price inflation than would result from staggered nominal wages alone.

The FRB model explicitly models potential output as a function of the labor force, crude energy use, and a composite capital stock, using a three-factor Cobb-Douglas production technology. The equilibrium output price is a markup over a weighted average of the productivity-adjusted wage rate and the domestic energy price. The specification of wage and price dynamics follows the generalized adjustment cost framework used in the FRB IS block. Wage inflation depends on its own (three) lagged values, expected future growth in prices and productivity, and a weighted average of expected future unemployment rates; price inflation depends on its own (two) lagged values, expected future changes in equilibrium prices, and expected future unemployment rates. In addition, both wages and prices error-correct to their respective equilibrium levels. As in the other models, a vertical long-run Phillips curve is imposed in estimation.

Unlike the other three models, the FRB model contains a detailed accounting of various categories of income, taxes, and stocks, an explicit treatment of labor markets, and endogenous determination of potential output. Long-run equilibrium in the FRB model is of the stock-flow type; the income tax rate and real exchange rate risk premium adjust over time to bring government and foreign debt-to-GDP ratios back to specified (constant) levels.

Foreign Sector. Neither FM nor MSR explicitly include foreign variables; in contrast, both TAYMCM and the full FRB staff model include detailed treatments of foreign variables. TAYMCM features estimated equations for demand components and wages and prices for the other G-7 countries at about the level of aggregation of the U.S. sector. The full FRB staff model includes a total of 12 sectors (countries or regions) which encompass the entire global economy. Because of the size of the model, the cost of solving and computing the moments of the full FRB model is prohibitive. Preliminary investigations using TAYMCM suggest that the characteristics of optimal U.S. monetary policies are not greatly affected by the precise specification of the foreign sector.[10] Based on these results and a need to economize on computational costs, we replaced the full set of equations describing foreign countries in the FRB staff model with two simple reduced form equations for foreign output and prices. For the remainder of the paper, we refer to this simplified version of the model as the FRB model.[11]

Dynamic Properties. We now turn to the basic dynamic properties of output and inflation--as measured by the unconditional autocorrelations--in the four models. Because output and inflation dynamics are sensitive to the specification of monetary policy, we begin by specifying a baseline policy rule used in each model for comparison purposes. For this purpose, we use the following interest rate reaction function, which was estimated using quarterly U.S. data over the sample period 1980Q1 - 1996Q4:

$$\mathbf{r}_t = \underset{(0.0036)}{-0.0042} + \underset{(0.07)}{0.795}\,\mathbf{r}_{t-1} + \underset{(0.13)}{0.625}\,\pi^{(4)}_t + \underset{(0.26)}{1.171}\,y_t - \underset{(0.23)}{0.967}\,y_{t-1} + \mathbf{u}_t, \quad (1)$$

$$\bar{R}^2 = 0.925;\ \text{SER} = 0.010;\ \text{DW} = 2.50$$

where $\mathbf{r}_t$ is the federal funds rate, $\pi^{(4)}_t$ is the four-quarter moving average of the inflation rate, and y_t is the current output gap.[12] Clarida, Gali and Gertler (1997a) and others argue that this period was characterized by a fairly stable policy regime that differed substantially from that of the 1960s and 1970s. This estimated policy rule features a relatively large coefficient on the lagged funds rate and a fairly aggressive response to increases in inflation and output gaps. Furthermore, the pattern of coefficients on the output gap and its lag suggest that policy not only responded to the level of output but also its recent growth.

We use the Anderson and Moore (1985) implementation of the Blanchard and Kahn (1980) method to solve for the VAR representation of the stable solution of the models, and we compute the unconditional moments of model variables analytically.[13] The inflation autocorrelogram of each model under the estimated policy rule is depicted in the upper panel of **Figure 1**.[14] Not surprisingly, the FM and MSR models--which share the Fuhrer-Moore contracting specification known to deliver a high degree of inflation persistence--are characterized by highly persistent inflation. Inflation is somewhat less persistent in the FRB model. As seen clearly in the figure, TAYMCM displays the least degree of inflation persistence, with autocorrelations that fall below zero after only four quarters. Even when combined with some inertia in price markups, the staggered nominal wage contract specification in TAYMCM delivers relatively low inflation persistence.

The lower panel of **Figure 1** depicts the output gap autocorrelogram of each model. In the FM model, the output gap is extremely persistent and displays some "over-shooting" in that the autocorrelation turns negative after five years. The FRB model output gap displays somewhat less persistence and more over-shooting than the FM model. The MSR and TAYMCM models--based on similar aggregate demand specifications--share the feature of a relatively low degree of output gap persistence, although it is slightly higher in the MSR model because the monetary policy response to inflation causes some of the persistence in inflation to spill over to output.

To provide a more detailed comparison of the dynamic properties of aggregate demand across models, it is useful to consider the following regression equation:

(2) $$\Delta y_t = \delta\, y_{t-1} + \theta\, \Delta y_{t-1} + \phi\, (\mathbf{r}_{t-1} - E_{t-1}\, \pi_t) + \varepsilon_t\,,$$

where $\mathbf{r}_{t-1} - E_{t-1}\, \pi_t$ is the lag of the *ex ante* real federal funds rate. The parameter δ indicates the degree of persistence (speed of error-correction) of the output gap; the parameter θ indicates the extent to which the output gap exhibits a short-run accelerator effect; and the parameter ϕ indicates the sensitivity of aggregate demand to a change in the short-term real interest rate. We use this simple specification to compare the basic properties of output dynamics between the models and to the data.

Table 2 indicates the asymptotic values of δ, θ, and ϕ for each model computed using the unconditional moments of each model and the estimated coefficients and standard errors obtained from fitting equation (2) using U.S. data over the sample period 1966Q1 - 1995Q4. Although the MSR model and TAYMCM have roughly similar output autocorrelograms, the table shows that these models actually imply very different behavior for the output gap. In particular, the output gap in TAYMCM error-corrects to zero rapidly and displays essentially no accelerator effect, whereas the output gap in the MSR model error-corrects more gradually and displays a strong accelerator effect. The FM and FRB models imply roughly similarly low rates of error-correction, while the accelerator effect in FM is nearly twice as strong as in the FRB model. Finally, the coefficients on the real short-term interest rate are similar across the four models, with FM displaying the least real interest rate sensitivity of output.

3. Policy Frontiers for Simple and Complicated Rules

This section outlines the objective function and constraints used in determining the inflation-output volatility frontier of each model for a given specification of the policy rule. Then we analyze the properties of these frontiers for several alternative specifications: rules in which the federal funds rate responds to only three variables (the current output gap, a moving average of the inflation rate, and the lagged funds rate), more complicated rules that incorporate a larger number of observed state variables, and rules that incorporate model-based forecasts of the output gap and inflation rate. Finally, we consider the extent to which these results are sensitive to a one-quarter delay in information.

Objective Function and Constraints. We assume that the interest rate rule is chosen to solve the following optimization problem:

(3)
$$\min_{\mathbf{f(z)}} \quad \lambda \; \mathrm{Var}(\mathbf{y}) + (1-\lambda)\, \mathrm{Var}(\pi)$$
$$\text{s.t.} \quad \mathbf{r} = \mathbf{f(z)}, \quad \mathbf{z} \in \mathbf{x},$$
$$\mathbf{x} = \mathbf{A(f)}\,\mathbf{L}\,\mathbf{x} + \mathbf{B(f)}\,\mathbf{e},$$
$$\mathrm{Var}(\Delta \mathbf{r}) \leq \mathbf{k}^2 .$$

where Var(s) is the unconditional variance of variable **s**. The weight $\lambda \in [0,1]$ indicates the policymaker's relative preferences for minimizing output and inflation volatility. The policy rule **f** is a time-invariant linear function of a specified set of variables **z** (e.g., the output gap and the inflation rate) that comprise a subset of **x**, the set of all variables in the model. The disturbance vector **e** is assumed to be serially uncorrelated with mean zero and finite second moments. The transition matrices **A(f)** and **B(f)** describe the unique saddle-path solution to the model, and **L** signifies the lag operator. Finally, we use the constraint on the variance of the first-difference of the funds rate as a way of controlling for the different amounts of interest rate volatility generated by different rules. Each policy frontier shown in this paper is constructed using a specified value of the upper bound, **k**.

Our approach to portraying policy frontiers differs slightly from that commonly found in the literature, where interest rate volatility is included in the objective function and each frontier is drawn using a different weight on interest rate volatility. The standard approach combines information on model-imposed constraints on policy with policymakers' preferences regarding funds rate volatility. We prefer to maintain the strict distinction between policy constraints--given by the model--and preferences. In this paper, each frontier represents the best obtainable combinations of output and inflation variability for a specified level of funds rate volatility.

We use the estimated rule given in equation (1) as a benchmark for funds rate volatility. For each model, we impose the estimated rule and compute the standard deviation of the one-quarter change in the funds rate. The outcomes from this experiment provide reference points for comparisons across the different models. We refer to frontiers constructed under the restriction on funds volatility that results from the estimated rule as E-frontiers. This normalization is needed because each model is estimated over different sample periods and generates quantitatively different distributions for the endogenous variables. For example, the moments for the FRB and TAYMCM models depend in part on shocks from the 1970's--a period of relative economic upheaval--while the shocks for the FM and MSR models are from the relatively tranquil 1980's and early 1990's. This difference in samples implies significant differences in interest rate volatility across the four models for a given policy rule.

Simple Policy Rules. We start by considering 3-parameter rules in which the federal funds rate $\mathbf{r}_t$ is determined as a linear function of the current output gap, y_t, the four-quarter average inflation rate, $\pi^{(4)}_t$, and the lagged funds rate, $\mathbf{r}_{t-1}$:

$$\mathbf{r}_t = \rho\,\mathbf{r}_{t-1} + (1-\rho)(\mathbf{r}^* + \pi^{(4)}_t) + \alpha(\pi^{(4)}_t - \pi^*) + \beta\, y_t \qquad (4)$$

where **r*** is the equilibrium real rate, and π^* is the inflation target (assumed to be constant throughout this paper).[15]

The solid lines in **Figure 2** depict the 3-parameter E-frontier of each model. As expected, the frontier is convex to the origin, with truncated vertical and horizontal asymptotes as the objective function in equation (3) switches from a concern only for inflation stabilization ($\lambda = 0$) toward one concerned only with output stabilization ($\lambda = 1$). Each panel of **Figure 2** also indicates the relative performance of the estimated rule, denoted by the letter "E." Because the estimated rule generates the same amount of funds rate volatility as the 3-parameter E-frontier of each model, comparison of the estimated rule to the policy frontier is straightforward. The estimated rule performs appreciably worse than the optimal 3-parameter rules for MSR, FRB, and TAYMCM, despite the fact that the estimated rule incorporates an additional variable (the lagged output gap). As discussed below, the optimal value of ρ for these three models is substantially higher than the estimated value of ρ in equation (1).

The coefficient values for optimal 3-parameter rules are shown in **Figure 3**. As the coefficients on inflation decrease, the standard deviation of inflation increases in all four models, as one would expect. The output gap coefficients vary noticeably less along the policy frontier. The output gap coefficient in MSR is close to unity along the whole policy frontier. In the FRB model, the output gap coefficient is smaller for rules corresponding to very high values of λ, but is otherwise quite close to unity. The output gap coefficient in TAYMCM ranges between 0.6 and 1.5 along the policy frontier. Finally, the optimal output gap and inflation coefficients for the FM model are much smaller than for the other three models.

The key result to be noted from **Figure 3** is that the 3-parameter policy frontiers of all four models are associated with rules in which the coefficient ρ on the lagged funds rate is very close to unity. In particular, the parameter ρ takes values in the range of {0.84, 0.95} for the FM model; {1.0, 1.1} for the MSR model; {0.96, 1.03} for the FRB model;[16] and {0.94, 0.98} for TAYMCM. Thus, the relatively poor performance of the estimated reaction function in the latter three models can be attributed mainly to the fact that the estimated value of $\rho = 0.8$ in equation (1) is substantially smaller than the optimal value of ρ for these models.

To examine this feature more closely, we consider the class of first-difference rules, which are a special case of the 3-parameter rules described by the previous equation:

$$(5) \qquad \mathbf{r}_t = \mathbf{r}_{t-1} + \alpha(\pi^{(4)}_t - \pi^*) + \beta y_t$$

For three of the four models (MSR, FRB, and TAYMCM), the E-frontier for first-difference rules is virtually identical to the 3-parameter E-frontier; i.e., imposing the constraint $\rho = 1$ is essentially costless in these models. For the FM model, first-difference rules are associated with slightly higher output and inflation volatility compared with 3-parameter rules, as seen by comparing the dashed and solid lines in the upper-left panel of **Figure 2**.

Finally, the lower-left panel of **Figure 2** shows that stabilization performance improves substantially in the FRB model if the simple policy rule is expressed in terms of $\pi^{(12)}_t$ rather than $\pi^{(4)}_t$; i.e., a three-year moving average of inflation instead of a one-year moving average. This feature differs across the four models: for the FM and MSR models, using a longer moving average of inflation (e.g., $\pi^{(8)}_t$ or $\pi^{(12)}_t$) provides relatively small improvements in terms of output and inflation volatility, while $\pi^{(4)}_t$ works best in the TAYMCM model. Thus, in the remainder of the paper, we use $\pi^{(12)}_t$ in computing the frontiers of simple rules for the FRB model, but continue to use $\pi^{(4)}_t$ for the other three models.

Complicated Policy Rules. In the simple rules described by equations (4) and (5), the funds rate is adjusted in response to only three variables: the current output gap, a moving average of the inflation rate, and the lagged interest rate. In practice, of course, central banks use much more information in making policy decisions. Furthermore, optimal control theory suggests that a policy rule should respond to all available information about the state of economy, that is, to all the state variables of the specific economic model under consideration. Thus, we investigate the extent to which complicated rules that respond to expanded subsets of the state variables can generate substantially lower output and inflation volatility.

Due to computational costs, our analysis of this issue focuses on the two smaller models, FM and MSR, for which we can investigate rules that include all observed state variables. The

FM model contains eight such variables (the current values of the output gap and inflation rate, two lags of the output gap, three lags of the inflation rate, and the lagged interest rate) while the MSR model contains 20 observed state variables. Results from TAYMCM for rules with up to six parameters confirm the findings we obtain from the two smaller models.

As seen in the top panel of **Figure 4**, 8-parameter rules that respond to all observable state variables in the FM model provide only negligible improvements in output and inflation stability beyond the optimal 3-parameter rules. Although the inflation and output variances do not change noticeably, the response coefficients in the more complicated rules differ quite a bit from the response coefficients in comparable 3-parameter rules. For example, the more complicated rules respond much more aggressively to inflation in the current quarter than inflation in the preceding 3 quarters.

Because demand is disaggregated in the MSR model, additional gains in performance might be expected by augmenting the 3-parameter rule in equation (4) to include consumption, fixed investment, inventory investment, and government spending. We find noticeable, but admittedly still moderate, improvements in output and inflation stability when moving from 3 to 8-parameter rules that include lags of the output gap and inflation rate, and again when moving from 8 to 12 parameters that include current values of the individual components of aggregate demand. The lower panel of **figure 4** shows the 3-parameter frontier and the 12-parameter frontier of the MSR model, while the 8-parameter frontier is omitted. As in the previous case, the coefficients of these complicated rules are quite different from simple rules, particularly the coefficients on quarterly inflation rates and the components of aggregate demand. Finally, although MSR contains other observed state variables, including these variables in the policy rules did not generate noticeable improvements in output and inflation stability beyond the 12-parameter frontier.

We conclude from this analysis that small improvements in output and inflation stability may be possible by including more variables (i.e., more information about the state of the economy) in the policy rule. Of course, such benefits may be offset by the lower degree of transparency associated with complicated policy rules. Furthermore, Section 5 will provide evidence that complicated policy rules are somewhat less robust to model uncertainty compared with optimal simple rules.

Rules with Model-Based Forecasts. Simple policy rules that incorporate model-based forecasts of the output gap and inflation rate implicitly respond to all the observed states in the model but remain parsimonious in terms of the number of free parameters in the rule. We have already noted that small reductions in output and inflation volatility can be obtained using complicated rules that respond to a large number of observed state variables. Now we analyze the extent to which these performance gains can be achieved by simple rules that incorporate forecasts of the output gap and inflation rate. In this analysis, we assume the forecasts are model-consistent and are known to the public.

We find three clear results. First, when we augment the class of 3-parameter rules to allow policy to respond additionally to one-quarter and two-quarter inflation forecasts, we find very little improvement in performance relative to optimal rules based only on current and lagged variables. Second, we consider 3-parameter rules in which the current output gap is replaced by model-based forecasts of the future output gap at various horizons. In all four models, rules that respond to the current output gap stabilize output and inflation more effectively than rules that use model-based forecasts of the output gap. Third, we consider 3-parameter rules in which the inflation variable is defined as an average of model-based forecasts as well as current and lagged inflation. In two models (FM and MSR), we find that the optimal inflation variable is an average of current and lagged inflation rates. In the other two models (FRB and TAYMCM), an average of current, lagged, and up to two leads of inflation yields a small improvement in performance.

Information Lags. In the preceding analysis, we have side-stepped one potentially important issue: policymakers may not have full knowledge of the current state of the economy, but instead must act upon data that comes in with a lag of weeks, months, or even longer. As McCallum (1997) has emphasized, policy rules must be *operational* in the sense that they can be implemented by policymakers in real time. In this paper, we do not examine the implications of using mismeasured data, but instead focus on the impact of information lags.[17]

A one-quarter information lag probably provides an accurate representation of the time delay with which information becomes available to policymakers. The first "advance" release of quarterly NIPA data occurs within one month of the end of the quarter, and monthly labor market and consumer price index (CPI) data are available by the middle of the following month. Other weekly and monthly data become available with short lags. To make the contrast to the results based on complete current information as stark as possible, we restrict ourselves to policies that depend only on lagged inflation, output gaps, and interest rates. We do not consider rules that incorporate current-quarter forecasts based on lagged information; if anything, this introduces an additional upward bias in our assessment of the true costs associated with informational lags.

Figure 5 compares the frontiers using current and lagged variables. The imposition of lagged variables in the rule imposes tiny costs in terms of stabilization in the FM and FRB models, and relatively small costs in the MSR and TAYMCM models. The characteristics of well-performing rules are also essentially unchanged. Evidently, a one-quarter information lag does not inhibit effective inflation and output stabilization in the models we consider here.

The reason for these small costs is that inflation and output are highly persistent in all four models and thus lagged inflation and output gaps are good proxies for the current ones. Still, the first-order autocorrelations are less than unity and the main differences between the models arise from the differences in output persistence across models. For the MSR and TAYMCM models, which display relatively low output persistence (first-order autocorrelation under first-difference rules of about 0.75 and 0.5, respectively), the cost of using lagged output gaps is larger than for the FM and FRB models with relatively high output persistence (0.95 and 0.85 first-order autocorrelations under first-difference rules, respectively). In the case of inflation, any effects on performance due to the use of lagged variables is further dampened by the fact that frontier rules depend on four- to twelve-quarter moving averages of inflation. The marginal impact of the current inflation rate is thus relatively small, so the shift in timing has little effect on the stabilization properties of frontier rules.

4. Comparison of Alternative Simple Rules

In this section, we focus in greater detail on the properties of 3-parameter rules of the form given in equation (4). This class of policy rules nests "level" rules such as those considered by Henderson and McKibbin (1993) and Taylor (1993), in which the lagged funds rate coefficient $\rho = 0$. We shall use the term "interest rate smoothing" to refer to rules in which this coefficient is substantially larger than zero, as in partial adjustment rules ($0 < \rho < 1$) and first-difference rules (ρ=1). As shown previously in **Figure 3**, the optimal value of ρ even exceeds unity in certain cases, but is never quite as large as the optimal values obtained by Rotemberg and Woodford (1997). Of course, level rules also tend to induce persistence in the funds rate, because output and inflation exhibit persistence in all four models.

Table 3 indicates the values of α, β, and ρ for six different rules in this class. Rules "A" and "B" are first-difference rules taken from the policy frontier of the FRB model. These rules correspond to values of $\lambda = 1/4$ and $\lambda = 3/4$, respectively (i.e., the weight on output volatility relative to inflation volatility) in the objective function given in equation (3). Rule "T" is the rule proposed by Taylor (1993a). Rule "T2" is a modified version of Taylor's rule in which the coefficient on the output gap has been doubled. Rules "V" and "W" are optimal policy rules for the dynamic general equilibrium model analyzed by Rotemberg and Woodford (1997).

Table 4 clearly shows that rules T and T2, in which the *level* of the funds rate responds to the output gap and inflation rate, are dominated by rules like A and B, where the *first-difference* of the funds rate responds to the output gap and inflation rate. Rules V and W exhibit very small responses to the output gap, and therefore also do relatively poorly at stabilizing output and inflation in the four models studied here. Even if policymakers only care about stabilizing inflation (i.e., $\lambda = 0$), the output gap coefficient should be substantially larger than that of rules V and W, because the current output gap is an important leading indicator for the inflation rate.

Figure 6 provides further information on the benefits of interest rate smoothing in comparison with level rules. The dotted line indicates the E-frontier associated with level rules; i.e., the inflation-output volatility frontier for rules having the form of equation (4) with $\rho = 0$, under the constraint that the standard deviation of $\Delta \mathbf{r}$ does not exceed that generated by the estimated rule in equation (1). The solid and dashed lines show the E-frontiers for 3-parameter rules and first-difference rules, as previously depicted in **Figure 2**. As previously noted, the E-frontiers for 3-parameter rules and first-difference rules are virtually identical for three of the models, while the optimal 3-parameter rules associated with the FM model incorporate a high degree of interest rate smoothing but with values of ρ noticeably smaller than unity.

Figure 6 reveals very substantial gains from interest rate smoothing in all four models. In fact, these are the largest gains we have found among all the permutations of simple policy rules that we have investigated.[18] In the FRB model, for example, using a first-difference rule instead of a level rule can reduce the standard deviation of output by a full percentage point.

Although empirical evidence reveals a pattern of interest rate smoothing in many industrial countries and is a property of the estimated U.S. interest rate rule in equation (1), the normative case for interest rate smoothing has remained much less clear.[19] Lowe and Ellis (1997) have recently surveyed the literature and summarized several considerations that tend to favor interest rate smoothing.[20] One argument, advanced by Goodfriend (1991) and others, is particularly relevant to our analysis: smooth changes in the short-term interest rate provide greater control over long-term interest rates and thereby greater control over aggregate demand and inflation. This rationale is explicitly captured in the four rational expectations models considered here: in each model, monetary policy stabilizes output and inflation mainly through its influence on the long-term real interest rate, which is determined as a weighted average of current and expected future short-term rates.[21] Thus, since the federal funds rate is more persistent under the smoothing rule compared with the level rule, a given initial adjustment of the federal funds rate induces a larger movement in the long-term bond rate and thereby achieves more rapid stabilization of output and inflation.

To evaluate the role of this mechanism in explaining the superior performance of interest rate smoothing rules, we conduct counterfactual experiments in each of the two smaller models, FM and MSR. In particular, we shorten the duration of the term structure equation, and compute new constrained output/inflation volatility frontiers based on rules with three parameters. In the FM model, we replace the 30-year bond rate with the current short-term rate in the IS curve. We find that the range of values of ρ along the frontier declines from [0.85,0.95] to [0.56,0.8]. In the MSR model, we replace the eight-quarter duration bond with the current funds rate in the consumption and investment equations and find that the range of coefficients on the lagged federal funds rate along the frontier declines from [1.0 ,1.1] to [0.75,1.0].

Thus, in both models, the optimal coefficient on the lagged funds rate is significantly reduced when the duration of the long rate is shortened, suggesting that the ability to create large movements in the long-term bond rate with small but persistent movements in the short rate is one of the principal explanations for the superior performance of interest rate smoothing rules in these forward-looking models.

However, because we compare frontiers for inherently second-best policy rules, that is, we constrain interest rate volatility and consider simple rules, we conjecture that the following three factors also play a role in generating the superior performance of interest rate smoothing: **(i)** the hypothesized preference for low volatility in the first difference of the interest rate may bias our results in favor of rules with a large coefficient on the lagged funds rate; **(ii)** output dynamics tend to exhibit overshooting under rules with $\rho \geq 1$, which can help stabilize output in terms of variances due to the forward-looking nature of the spending equations in our models; **(iii)** by including the lagged funds rate, the policy rule implicitly responds to lagged as well as current values of the output gap and inflation rate, and thus incorporates more information about the state of the economy.

We first consider the relationship between the optimal value of ρ and the preference for funds rate volatility implicitly imposed in drawing the frontiers. For each model, **Figure 7** shows three frontiers with different restrictions on funds rate volatility. In each case the baseline E-frontier is shown as a solid line. Using frontier rules as a guide, the stabilization gains from increased funds rate volatility are evidently rather small. **Table 5** shows typical values of the standard deviation of the *level* of the funds rate generated by the policies that underlie the frontiers shown in the figure. In each case, outcomes corresponding to the baseline E-frontier are given in the first two columns. Except for the MSR model, relaxing the constraint on funds rate volatility much beyond that implied by the E-frontier entails so much volatility of the level of the funds rate that the optimal policy rule would regularly dictate negative nominal interest rates in an environment of reasonably low steady-state nominal interest rates (steady state inflation plus real interest rate).

Qualitatively, all four models exhibit a pattern in which allowing higher interest volatility is associated with smaller values of ρ for the 3-parameter rules on the policy frontier. However, the quantitative results differ somewhat across models. In the FM model, the reduction in ρ is particularly pronounced: doubling the upper bound on SD(Δ**r**) causes the range of values for ρ to drop from [0.85,0.95] to [0.75,0.92]. In the other three models, relaxing the funds rate volatility constraint leads to somewhat smaller reductions in the optimal value of ρ. In the FRB model, doubling SD(Δ**r**) reduces the optimal value of ρ by about 0.07 on average, from a range of [0.96,1.02] to [0.93,0.96]. In TAYMCM and in MSR, doubling SD(Δ**r**) only reduces the optimal value of ρ by about 0.03. Evidently, even with relatively high interest rate volatility, a high degree of interest rate smoothing is preferred in these models.

Next, we consider the argument that interest rate rules with a high degree of interest rate smoothing reduce overall variability by generating secondary cycles, e.g., overshooting of output. **Figure 8** shows the stark contrast in the dynamic response of output to a demand shock under a level rule versus a 3-parameter rule with interest rate smoothing. Each rule corresponds to the policy frontier associated with the same level of interest volatility as that resulting from the estimated rule in equation (1). In FM, the demand shock is a positive shock of one standard deviation to the IS curve. In MSR, the composite demand shock is the sum of one-standard-deviation shocks to consumption, fixed investment, inventory investment, and government spending. In both models, the smoothing rules substantially dampen the response of output in the first few quarters compared to the level rules and subsequently push output below potential for some time. Because the spending equations in both FM and MSR are forward-looking, these expected future movements in output, prices, and short-term interest rates play a role in dampening the initial impact of the aggregate demand shock. Of course, given the objective of minimizing the variance of output and inflation, there is a tradeoff between reducing the peak and increasing the extent of overshooting. Nevertheless, in both models overshooting resulting from policy with large values of ρ is preferred to a monotone reversion to equilibrium values; for example, the interest rate smoothing rule reduces the standard deviation of output by 15 percent in FM and by nearly 40 percent in MSR, compared with the level rule.

Finally, optimal control theory suggests that the optimal policy rule should take into account all available information, including lagged values of the output gap and inflation rate. A rule with a high value of ρ implicitly makes the current interest rate depend on the complete history of output and inflation, albeit in a very restricted way. Thus, the lagged funds rate may be serving as a simple proxy that permits additional information to be included in a suboptimal level rule. We test this explanation by computing frontiers for rules that include all observable state variables, and then checking whether these complicated rules are characterized by smaller lagged funds rate coefficients. Because of the high computational cost in conducting this experiment, we focus on the small models, FM and MSR. The policy frontiers associated with complicated rules for these models have already been shown in **Figure 4**. Here we simply note that rules which include all observed state variables call for somewhat smaller coefficients on the lagged funds rate. In the FM model, the range of values of ρ along the frontier is almost identical for the case of three-parameter rules as for rules that respond to all observable state variables, while in the MSR model, the range of values of ρ decline from [1.0,1.1] to [0.91,0.99] for rules that respond to all observed state variables.

5. Robustness to Model Uncertainty

To evaluate the extent to which simple policy rules are robust to model uncertainty, we take first-difference rules from the policy frontier of one model, and evaluate the performance of these rules in each of the other three models. In particular, we consider the first-difference rules A and B from the corresponding policy frontier of the FRB model; the parameter values for these rules are given in **Table 3**. In the FRB model, both rules generate the same standard deviation of the first-difference of the funds rate as that generated by the estimated rule given in equation (1). For each of the other models, we calculate the funds rate volatility associated with each of the two rules, and then we compute a separate policy frontier for each upper bound on funds rate volatility, using the class of 3-parameter rules given by equation (4). For example, the "Rule A" policy frontier for the MSR model is the 3-parameter policy frontier for rules with the same funds rate volatility that rule A generates in this model. For the FM model, rules A and B produce virtually identical amounts of funds rate volatility.

The results of this analysis are depicted in **Figure 9**, which shows that rules A and B provide reasonably efficient performance in stabilizing output and inflation in all four models. Conditional on the level of interest rate volatility implied by these rules, the coefficients of the rules are such that inflation and output volatility lie very near the 3-parameter policy frontiers of all models. Evidently, in terms of efficiently reducing the volatility of output and inflation, well-chosen simple rules are very robust to the type of model uncertainty encompassed by these four models. Nevertheless, while these results show that rules A and B are reasonably efficient, this figure does not indicate how well the rules perform in terms of specific values of λ.

Table 6 reports several summary statistics about the extent to which rules A and B are robust to model uncertainty. The fourth column reports the value of the objective function using the "true" value of λ (assumed to be 0.25 for rule A and 0.75 for rule B, which are the values used to determine these rules in the FRB model). The fifth column reports the absolute loss in terms of the objective function implied by following the specified rule (A or B) instead of the three-parameter optimal rule (with the same amount of interest rate volatility) for the specified value of λ. The last column reports the value of λ which would be consistent with the choice of rule A or B for the particular model; e.g., in the first row, the implicit value of $\lambda = 0.1$ means that policymakers with preferences described by $\lambda = 0.1$ would choose rule A in the FM model.

These results indicate that if a policymaker were to use the FRB model to choose a policy rule, but the real world were actually described by one of the other three models, the policy rule would generate slightly greater output volatility and slightly less inflation volatility compared with the preferences of the policymaker. In particular, the implicit values of λ associated with rules A and B are all smaller in the non-FRB models than the "true" values used in choosing rules A and B in the FRB model. However, while rules A and B are suboptimal, the loss in terms of the objective function, measured either in absolute or percentage terms, is quite small in the MSR model and in TAYMCM. The somewhat larger loss in the FM model occurs because the optimal 3-parameter rule for that model uses a coefficient below unity on the lagged funds rate.

Now we consider the extent to which complicated rules are robust to model uncertainty. In particular, we take two complicated rules (denoted by "P" and "Q") from the 12-parameter E-frontier of the MSR model, and determine the performance of these rules in the FRB model and in TAYMCM. Because rules P and Q cannot be implemented directly in the FM model (which does not explicitly treat the components of aggregate demand), we also take two rules (denoted by "R" and "S") from the 8-parameter E-frontier of the MSR model and evaluate the performance of these rules in the FM model. As above, the output and inflation variability of the complicated rules should be compared with simple rules that generate the same level of interest rate volatility. Thus, for each model, we calculate the funds rate volatility associated with each of the two complicated rules, and then we compute a separate policy frontier for each upper bound on funds rate volatility, using the class of 3-parameter rules given by equation (4).

As shown in the left panels of **Figure 10**, the more complicated rules lie fairly close to the 3-parameter policy frontiers of the FM and FRB models. In TAYMCM, however, the two 12-parameter rules are much less effective in stabilizing output and inflation than the optimal 3-parameter rules. Thus, while small improvements in output and inflation variability may be obtained by using complicated policy rules, these rules are somewhat less robust to model uncertainty compared with simple rules. As discussed in Section 2, the dynamic properties of output and inflation differ substantially across the four models. Thus, it is not very surprising that fine-tuning a complicated rule to one particular model may not be appropriate when policymakers are concerned about model uncertainty.

6. Conclusions

This paper has investigated the performance of policy rules across four structural macroeconomic models with rational expectations. Although the four models differ in many important respects (e.g., the level of aggregation, the specification of output and price dynamics, and the treatment of the foreign sector), the characteristics of effective policy rules are essentially the same. To stabilize inflation and output at reasonably low levels of interest rate volatility, the policy rule should respond to the current output gap and to a one- to three-year moving average of the inflation rate, and should incorporate a high degree of interest rate smoothing, that is, a coefficient near unity on the lagged funds rate. These results are essentially unchanged even if the policy rule is restricted to react to output and inflation data from the previous rather than the current quarter.

Interest rate smoothing provides the largest gains from any of the permutations of simple policy rules that we have investigated. Several factors contribute to this result: (i) smooth changes in the short-term interest rate provide control over long-term interest rates and thereby over aggregate demand and inflation at low cost in terms of funds rate volatility; (ii) constraining interest rate volatility as we do in constructing the frontiers favors interest rate smoothing; (iii) the lagged interest rate provides a measure of the existing state of the economy in models with output and inflation persistence; and (iv) with a very high degree of smoothing, such as that associated with first-difference rules, output tends to exhibit "overshooting", which is preferable to returning monotonically to potential under the standard deviation criteria employed here.

Simple rules derived from one model perform very well in the other three models; i.e., these rules are robust to model uncertainty within this class of models. For a given model, complicated rules perform only slightly better than simple ones, even when all observed state variables are incorporated in the rule. Furthermore, these rules are somewhat less robust to model uncertainty compared with well-chosen simple rules. Thus, fine-tuning a complicated policy rule to one specific model may not be advisable, because policymakers are faced with substantial uncertainty about the true structure of the economy as well as with competing views about the quantitative effects of alternative policy actions.

Finally, rules that incorporate forecasts of the output gap and inflation rate generally do not outperform optimal rules based on current and lagged variables. This result is related to that regarding complicated rules: even in large models with hundreds of state variables, three variables (the current output gap, a moving average of current and lagged inflation rates, and the lagged funds rate) summarize nearly all the information relevant to setting the federal funds rate efficiently.

Appendix: US. Monetary Policy under Alternative Foreign Policy Regimes

Our initial results indicate that within a fairly wide range of alternative foreign monetary policy assumptions (e.g., fixed money growth, interest rate rules), the specific foreign monetary policy regime appear to have only minor implications for the properties of a U.S. monetary policy rule. For example, consider the class of rules in which the first-difference of the federal funds rate responds to the current output gap and inflation deviation from target.

Table 7 provides information about the U.S. inflation-output volatility frontier under two alternative assumptions about foreign monetary policy: (a) each foreign G-7 country follows a fixed money growth rule; or (b) France, Germany, Italy, and the U.K. belong to a monetary union in which the European Central Bank adjusts interest rates in response to the European average output gap and average inflation deviation from target, while Canada and Japan independently follow similar rules. **Table 7** shows that in TAYMCM, neither the position of the U.S. inflation-output volatility frontier nor the coefficients of rules on the policy frontier are sensitive to this choice of foreign monetary policy assumptions. Impulse response functions generated using the full FRB staff model (cf. Levin et al. 1997) yield similar conclusions.

Based on these results, the results in this paper are based on a smaller version of the FRB staff model, referred to as the FRB model. In this model, foreign output, prices, *ex ante* long-term real interest rates, and the oil import price deflator are generated by simple autoregressive processes. The trade-weighted real exchange rate is determined by the differential between U.S. and foreign *ex ante* long-term real interest rates, with an endogenous risk premium that ensures a stable ratio of net external debt to nominal GDP.

NOTES

1. We use the term "model uncertainty" to refer to lack of knowledge about which model among a given set of alternatives provides the best description of the economy. For a particular model, we treat the estimated parameters as known with certainty. A very limited literature exists on the problem of conducting monetary policy under model uncertainty (Becker et al. 1986; Frankel and Rockett 1988; Holtham and Hughes-Hallett 1992; and Christodoulakis et al. 1993). Optimal policy under parameter uncertainty was investigated in the seminal paper of Brainard (1967), and was extended by the work of Kendrick (1982) and others; the more recent literature includes Balvers and Cosimano (1993) and Wieland (1996b, 1997).

2. MSR is a small macroeconometric model of the U.S. economy from 1980 to 1996, developed and used for research on monetary policy rules in the Monetary Studies Section at the Federal Reserve Board (e.g., Orphanides et al. 1997).

3. The literature on policy evaluation using traditional structural models is large and includes important contributions by Cooper and Fischer (1977) and Poole (1970). In recent papers, Fair and Howrey (1996), Ball (1997), and Rudebusch and Svensson (1998) derive optimal policies from traditional structural macroeconomic models.

4. Williams (1997) compares the characteristics of optimal policies under rational expectations and alternative assumptions regarding expectations formation using the FRB staff model.

5. Taylor (1979) and, more recently, Fuhrer (1997a), Svensson (1997a), and Tetlow and von zur Muehlen (1997), derive optimal policies in small rational expectations structural macro models.

6. This nonlinearity has been investigated in both of the smaller models used here. Fuhrer and Madigan (1997) conducted deterministic simulations of the Fuhrer-Moore model to assess the extent to which the zero bound prevents real rates from falling and thus cushioning aggregate output in response to negative spending shocks. Orphanides and Wieland (1997), using stochastic simulations of MSR, find that the effectiveness of monetary policy is significantly reduced at inflation targets below 1%. They find that the distortions due to the zero bound generate a non-vertical long run Phillips curve and result in higher inflation and output variability.

7. The demand side of this model is similar to Taylor (1993a) and Taylor and Williams (1993), while the wage-price sector is taken from Fuhrer (1997b).

8. The consumption equations are based on the life-cycle theory and include different marginal propensities to consume out of different categories of wealth (labor income, property income, stock market) reflecting the differing characteristics of the owners of these assets. See Brayton (1997b) for a discussion.

9. For FM we use the estimated parameters from Fuhrer (1997a). The MSR model uses the parameter estimates of Fuhrer (1997b), for which the implied response of inflation to output gaps is substantially larger than in Fuhrer (1997a) and more similar to the results of Fuhrer and Moore (1995).

10. For example, using TAYMCM, we computed optimal policies under two alternative assumptions regarding foreign monetary policies. In the baseline case, each foreign country is assumed to follow an independent constant growth rule for money; in the alternative case, the EU countries are assumed to follow a single currency policy described by a Taylor (1994) rule and Canada and Japan follow independent policies using the same Taylor rule specification. The optimal U.S. policy rules were almost identical across the two cases. Details are given in the Appendix.

11. This version of the model is typically referred to as FRB/US, while the full model is referred to as FRB/GLOBAL.

12. Inflation is measured using the chain-weighted GDP deflator and the output gap is based on estimates of potential output supplied by the Congressional Budget Office. This rule forms part of the MSR model and is taken from Orphanides and Wieland (1997).

13. We log-linearized FRB and linearized TAYMCM around sample means. Because the two models are nearly linear anyway, this linearization has virtually no effect on the second moments studied in this paper. On a Sun Ultra Enterprise 3000 computer (SPECfp_base95 = 8.45, about twice the speed of an Intel Pentium 233), it takes seconds to solve and compute moments for the Fuhrer-Moore and MSR models and about five minutes each for the FRB and Taylor models. The solution algorithm has been modified to use sparse matrix operations. The moment computations take advantage of the doubling algorithm described in Hansen and Sargent (1997).

14. Throughout the paper, inflation refers to the growth in the GDP price level for FM, MSR, and TAYMCM, and that of the personal consumption price index for FRB.

15. Throughout the analysis in this paper, we assume that the inflation target is high enough to avoid the distortions documented in Orphanides and Wieland (1997) for low inflation regimes.

16. As seen in **Figure 3**, along the vertical asymptote the level and first-difference rule frontiers overlap. For very low weights (0.0 - 0.05) on output variance in the policy objective function, the optimal value of ρ ranges from 0.5 to 0.85, but this coefficient quickly rises to nearly unity as the weight on output variance rises above 0.05.

17. Sources of mismeasurement relevant for policy rules include the noise in the data due to sampling and data imputation methods and imprecise estimation of potential output and the natural rate of unemployment. Orphanides (1997) studies the relevance of measurement problems for policy rules. Wieland (1997) examines optimal policy when the natural rate is unknown.

18. The beneficial impact of interest rate smoothing extends to models that fall outside the class of models we consider here, such as the models in Woodford and Rotemberg (1997) and Rebelo and Xie (1997), that explicitly incorporate optimizing behavior of representative agents.

19. Clarida, Gali and Gertler (1997a) document the practice of interest-rate smoothing for U.S. monetary policy.

20. For example, policymakers may dislike frequent reversals in interest rates, either because such changes make the policymaker look poorly informed and undermine confidence in the central bank as argued by Caplin and Leahy (1997), or because it is difficult to obtain broad-based political support for such changes in direction as suggested by Goodhart (1997). Furthermore, by avoiding large movements in interest rates the central bank can reduce financial market volatility and in doing so reduces the likelihood of instability when particular institutions incur large losses. Finally, the nature of the decision-making process may lead to caution. For example, Alan Blinder (1995), when Vice-Chairman of the Board of Governors, argued that uncertainty that policymakers have about the parameters of the underlying model justifies "stodginess" in monetary policy.

21. TAYMCM and FRB also include an explicit exchange rate channel of monetary policy and FRB includes a channel for wealth, which is negatively related to long-term real interest rates.

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Table 1

Comparison of Model Specifications

	FM	MSR	FRB	TAYMCM
IS Components	1	5	17	10
Price Variables	2	2	7	5
Labor Variables	0	0	4	0
Asset Prices	1	2	7	2
Foreign Variables	0	0	2 [2]	95
Wage and Price Dynamics	Staggered real wage contracts	Staggered real wage contracts	Generalized adjustment costs	Staggered nominal wage contracts
Duration of Bond	30 years	2 years	5 years [3]	2 years
Permanent Income Discount Rate	N/A	10% per quarter	7% per quarter	10% per quarter
Estimation Period	1982-94[1]	1980-96	1966-95	1971-86

Notes:

1. FM estimated over 1966:1-1994:1; residuals are computed from 1982:4-1994:1.
2. The full FRB model contains over 400 foreign variables; in the version of the FRB model used in this paper, these have been replaced by 2 equations for foreign output and prices.
3. The FRB model includes bonds with durations of 5, 10, and 30 years, as well as equity prices. The five year bond is used in computing the cost of capital for business equipment and consumer purchases of durables.

Table 2

Comparison of Output Dynamics

Coefficient	Models: FM	MSR	TAYMCM	FRB	U.S. Data 1966:1-1995:4
δ	-0.04	-0.17	-0.31	-0.05	-0.10 (0.03)
θ	0.49	0.45	-0.04	0.26	0.29 (0.08)
ϕ	-0.03	-0.05	-0.06	-0.04	-0.08 (0.03)

Note: This table considers the regression equation $\Delta y_t = \delta\, y_{t-1} + \theta\, \Delta y_{t-1} + \phi\, (r_{t-1} - E_{t-1}\pi_t) + \varepsilon_t$, where y_t is the output gap and $r_t - E_t\pi_{t+1}$ is the *ex ante* real federal funds rate. The first four columns report asymptotic values of δ, θ, and ϕ for each of the four models, while the final column reports the coefficient estimates and OLS standard errors of this regression equation using U.S. data for the sample period 1966Q1-1995Q4. For the data regression, expected inflation is proxied by the lagged inflation rate.

Table 3

Parameter Values of Alternative Simple Rules

Policy Rule	Parameters: α	β	ρ
A	1.3	0.6	1.0
B	0.8	1.0	1.0
T	0.5	0.5	0
T2	0.5	1.0	0
V	1.5	0.06	1.3
W	1.6	0.08	1.3

Table 4

Rule Comparison Table

Model	Rule	Standard Deviations SD(y)	SD(π)	SD(r)	SD(Δr)
FM	**A**	3.78	1.85	8.89	1.97
	B	2.37	2.45	7.71	1.83
	T	2.68	2.63	3.57	0.75
	T2	2.32	2.84	3.83	0.90
	V	21.2	7.13	27.2	4.38
	W	20.5	6.57	27.9	4.59
MSR	**A**	0.84	0.4	1.17	0.34
	B	0.58	0.53	1.33	0.48
	T	0.99	0.7	1.01	0.3
	T2	0.87	0.73	1.19	0.5
	V	1.95	0.41	1.31	0.19
	W	1.88	0.38	1.3	0.19
FRB	**A**	2.12	1.46	4.34	1.22
	B	1.41	1.65	4.5	1.24
	T	2.92	1.86	2.51	0.9
	T2	2.21	2.02	3.16	1.2
	V	6.32	1.55	4.67	1.11
	W	6.06	1.53	4.88	1.19
TAYMCM	**A**	2.33	1.73	4.78	1.71
	B	1.95	1.79	5.03	2.01
	T	2.89	2.58	4	1.58
	T2	2.55	2.36	4.35	2.41
	V	4.31	2.06	4.24	1.24
	W	4.26	2.02	4.47	1.33
U.S. Data (1980:1 - 1995:2)		2.4	2.1	3.7	1.3

Note: The U.S. data uses a measure of potential output provided by the Congressional Budget Office. Caution should be used in comparing the sample moments with those of each model, because the sample period involves an exceptionally high initial inflation rate, whereas the model-based moments represent deviations from a stochastic steady state.

Table 5
Volatility of Funds Rate Levels and Changes

	E Frontier		Alternative Frontier #1		Alternative Frontier #2	
Model	SD(Δr)	SD(r)	SD(Δr)	SD(r)	SD(Δr)	SD(r)
FM	1.0	5	2.0	7	3.0	9
MSR	0.6	1.5	1.2	2.3	1.7	2.9
FRB	1.2	4.5	2.4	7	3.7	9
TAYMCM	2.6	6	5.1	9	7.7	11

Table 6
Performance of Simple Rules Under Model Uncertainty

		Standard Deviations			Objective Function		
Model	Rule	y	π	Δr	Value	Loss	Implicit λ
FM	A	3.78	1.85	1.97	6.15	0.86	0.10
	B	2.37	2.45	1.83	5.72	1.28	0.40
MSR	A	0.84	0.40	0.33	0.30	0.01	0.15
	B	0.58	0.53	0.48	0.33	0.00	0.45
TAYMCM	A	2.33	1.73	1.71	3.61	0.17	0.10
	B	1.94	1.79	2.00	3.64	0.03	0.40

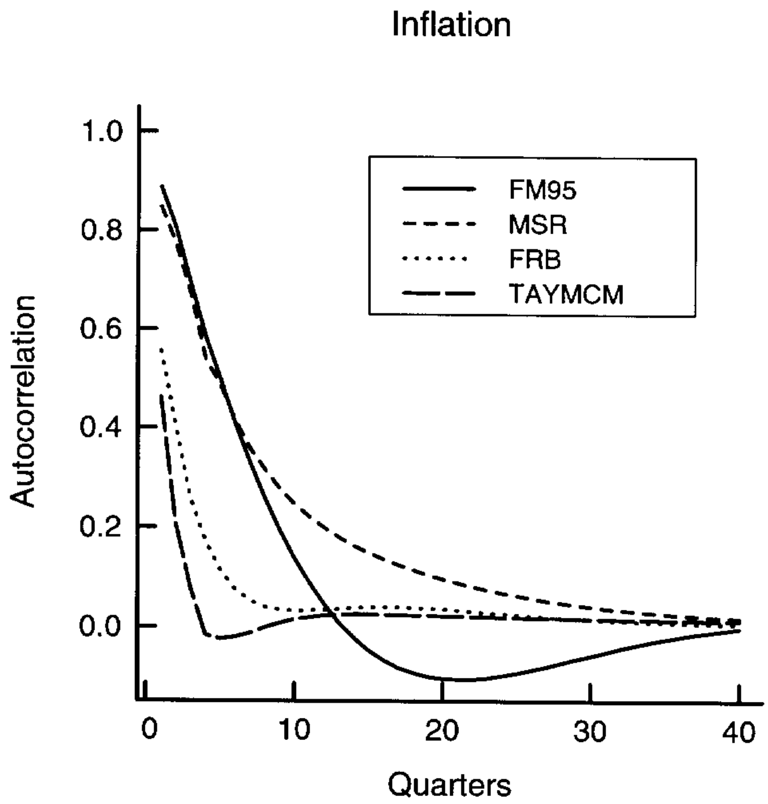
Table 7

Comparison of Foreign Monetary Policy Regimes in TAYMCM

	$\lambda = 0.1$		$\lambda = 0.25$		$\lambda = 0.5$	
	Fixed μ	**EMU**	**Fixed μ**	**EMU**	**Fixed μ**	**EMU**
SD(y_t)	2.12	2.13	1.83	1.83	1.75	1.75
SD(π_t)	1.73	1.73	1.80	1.80	1.84	1.84
α	1.17	1.16	1.40	1.40	1.46	1.46
β	1.92	1.92	0.96	0.96	0.48	0.48

Note: This table provides information on the U.S. inflation-output volatility frontier for interest rate rules of the form $\Delta \mathbf{r}_t = \alpha\, y_t + \beta(\pi_t - \pi^*)$, where $\mathbf{r}_t$ is the federal funds rate, y_t is the U.S. output gap, and $\pi_t - \pi^*$ indicates the deviation of U.S. inflation from the target rate. For $0 < \lambda < 1$, the policy frontier minimizes the objective function λ SD(y_t) + $(1-\lambda)$ SD($\pi_t - \pi^*$) subject to the constraint that SD($\Delta \mathbf{r}_t$) $\leq$ 2.57, where SD(z) indicates the unconditional standard deviation of z. Fixed μ and EMU are the two alternative foreign monetary policy regimes described in the text above.

Figure 1: Persistence Implications of Four Models



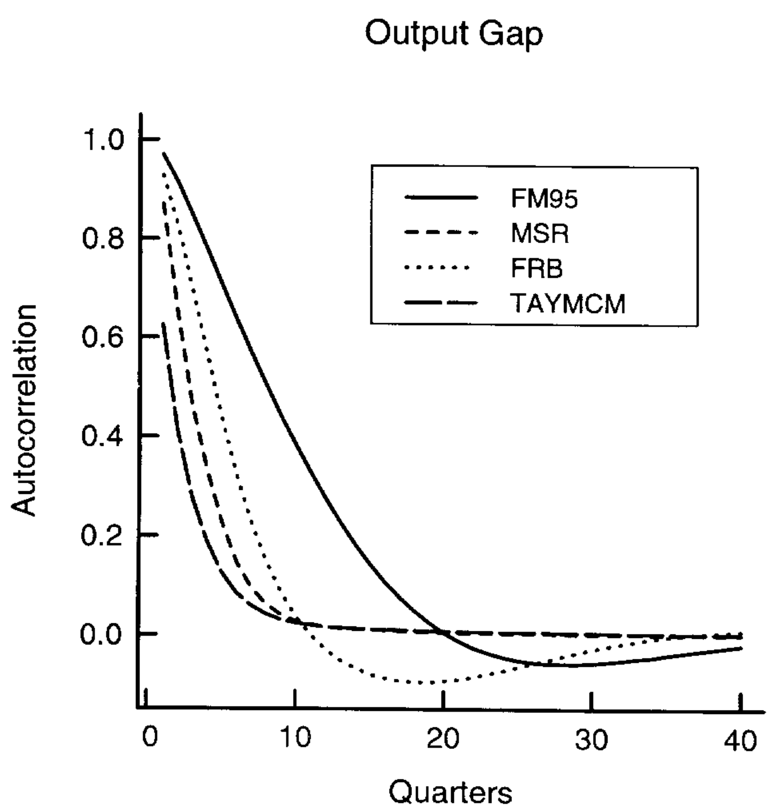


Figure 2: Policy Frontiers for 3-Parameter Rules

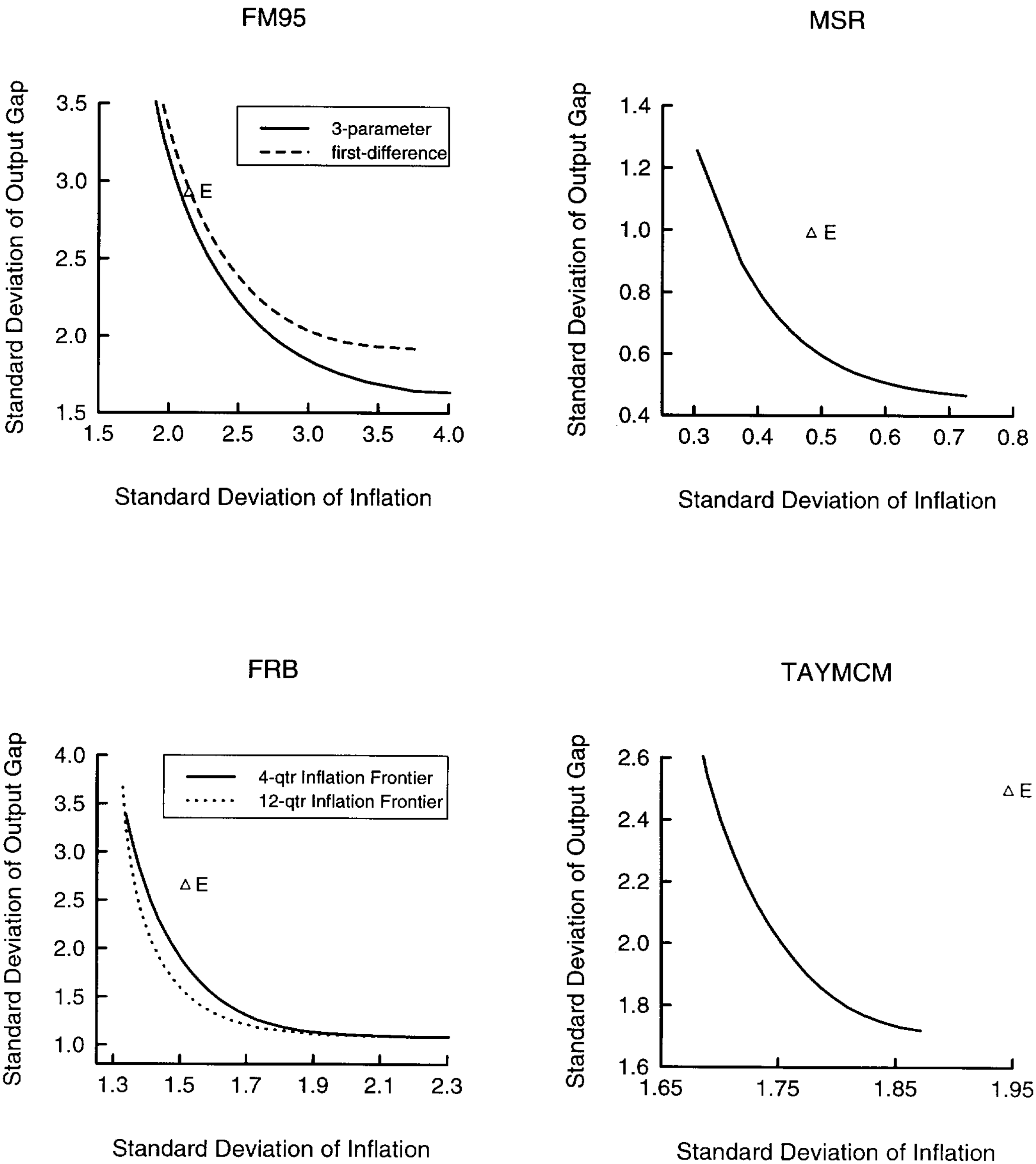


Figure 3: Coefficients of Optimal 3-Parameter Rules

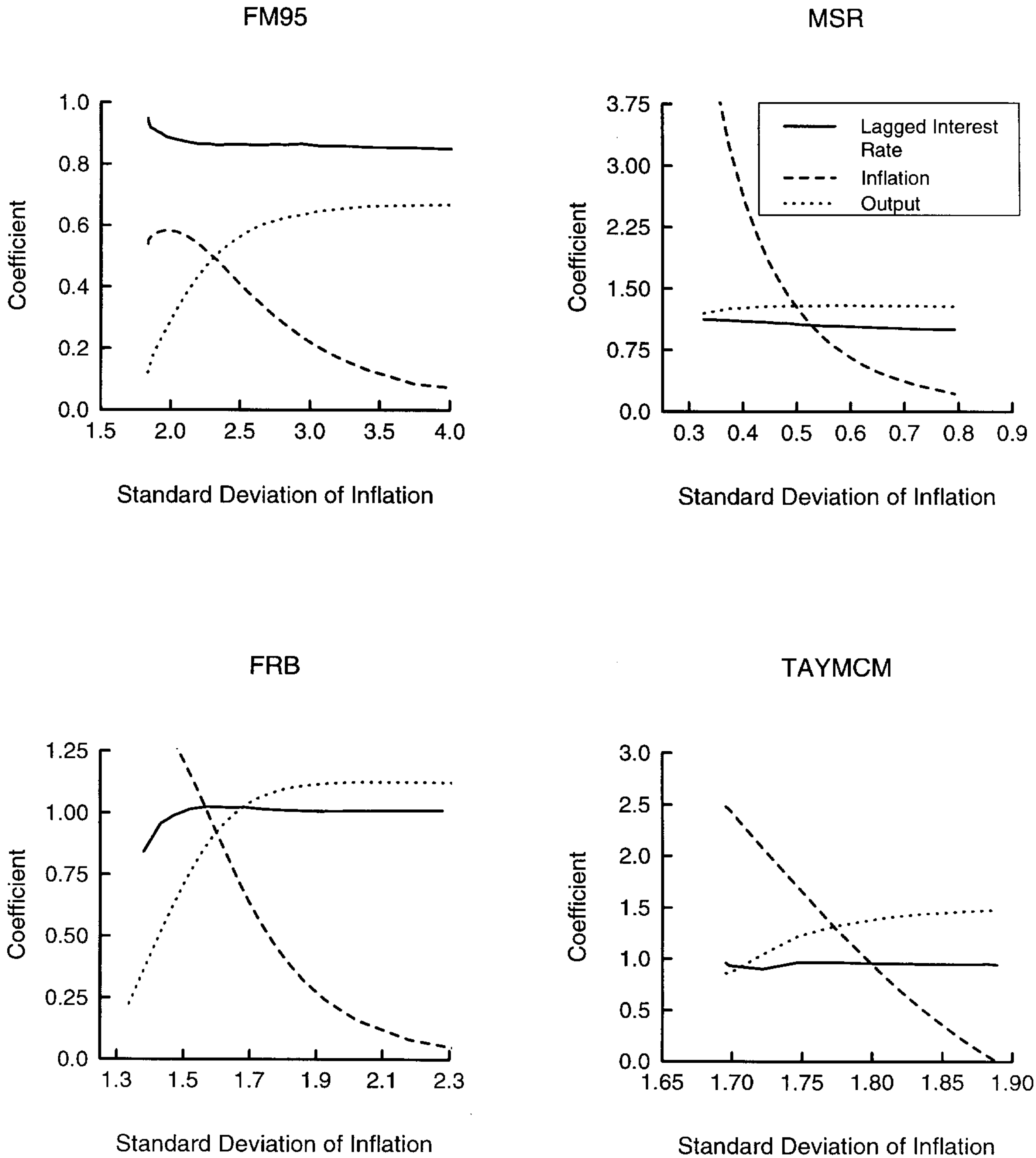


Figure 4: Policy Frontiers for Complicated Rules

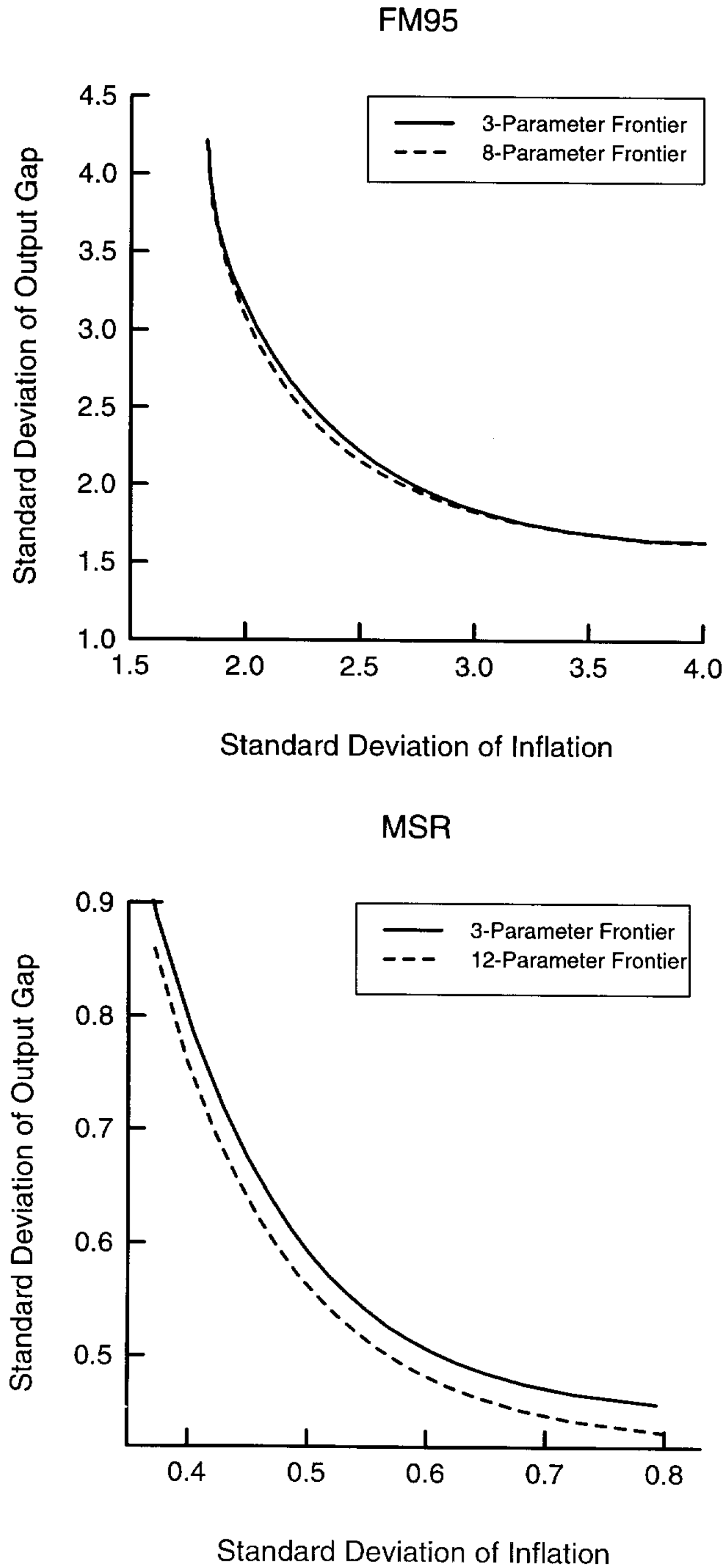


Figure 5: Implications of Using Lagged Information

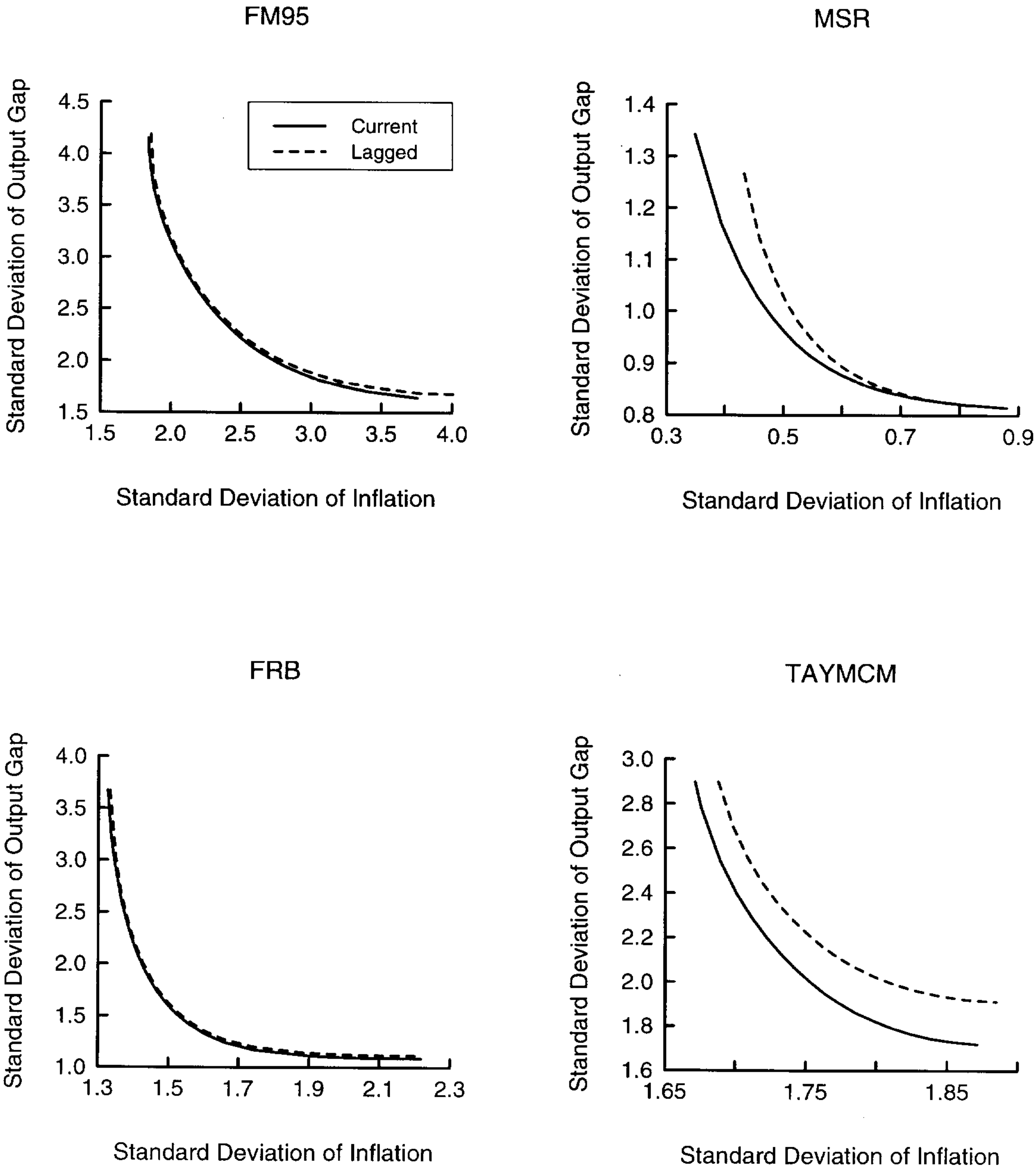


Figure 6: Comparison with Level Rules

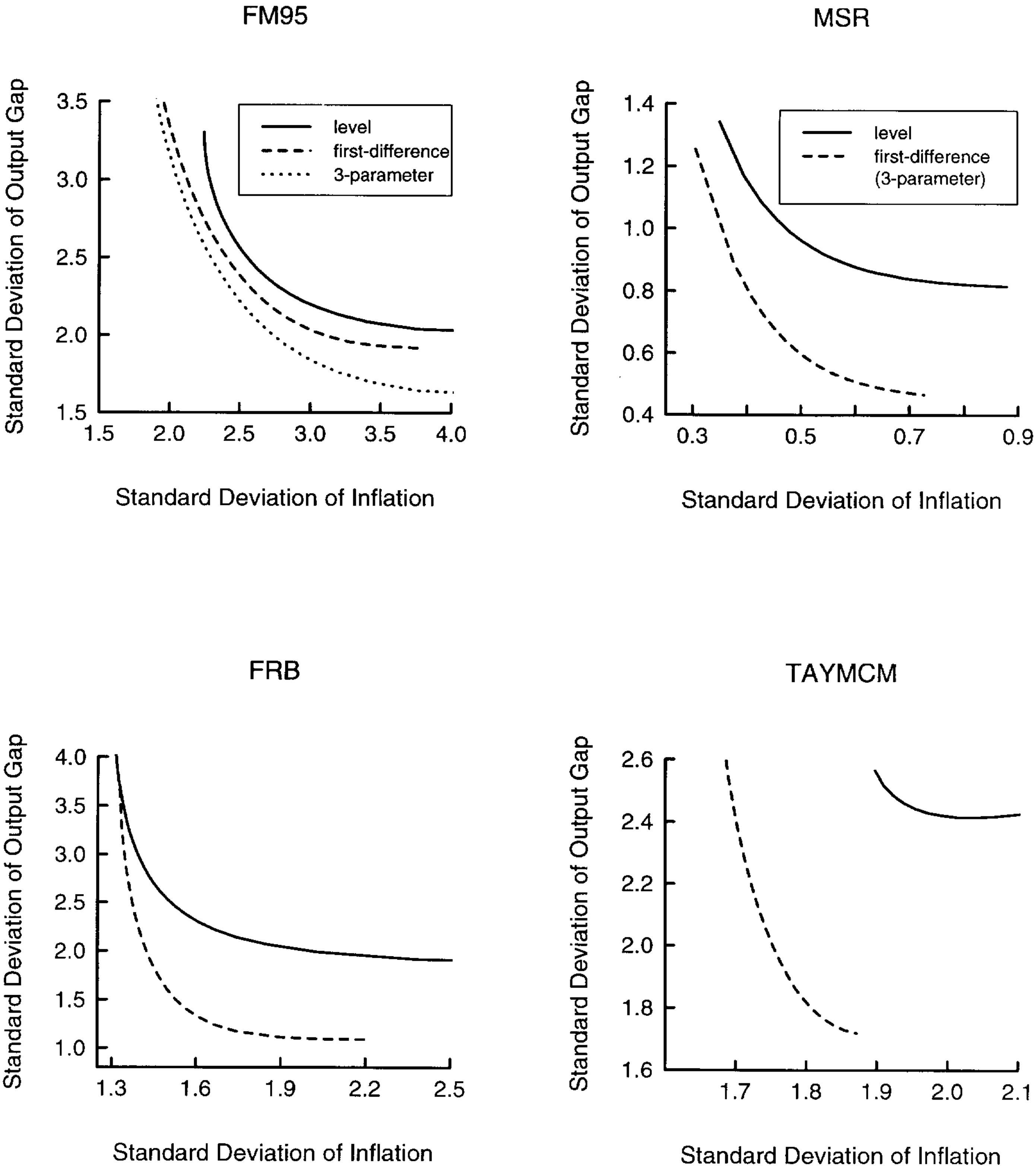


Figure 7: Alternative Constraints on Funds Rate Volatility

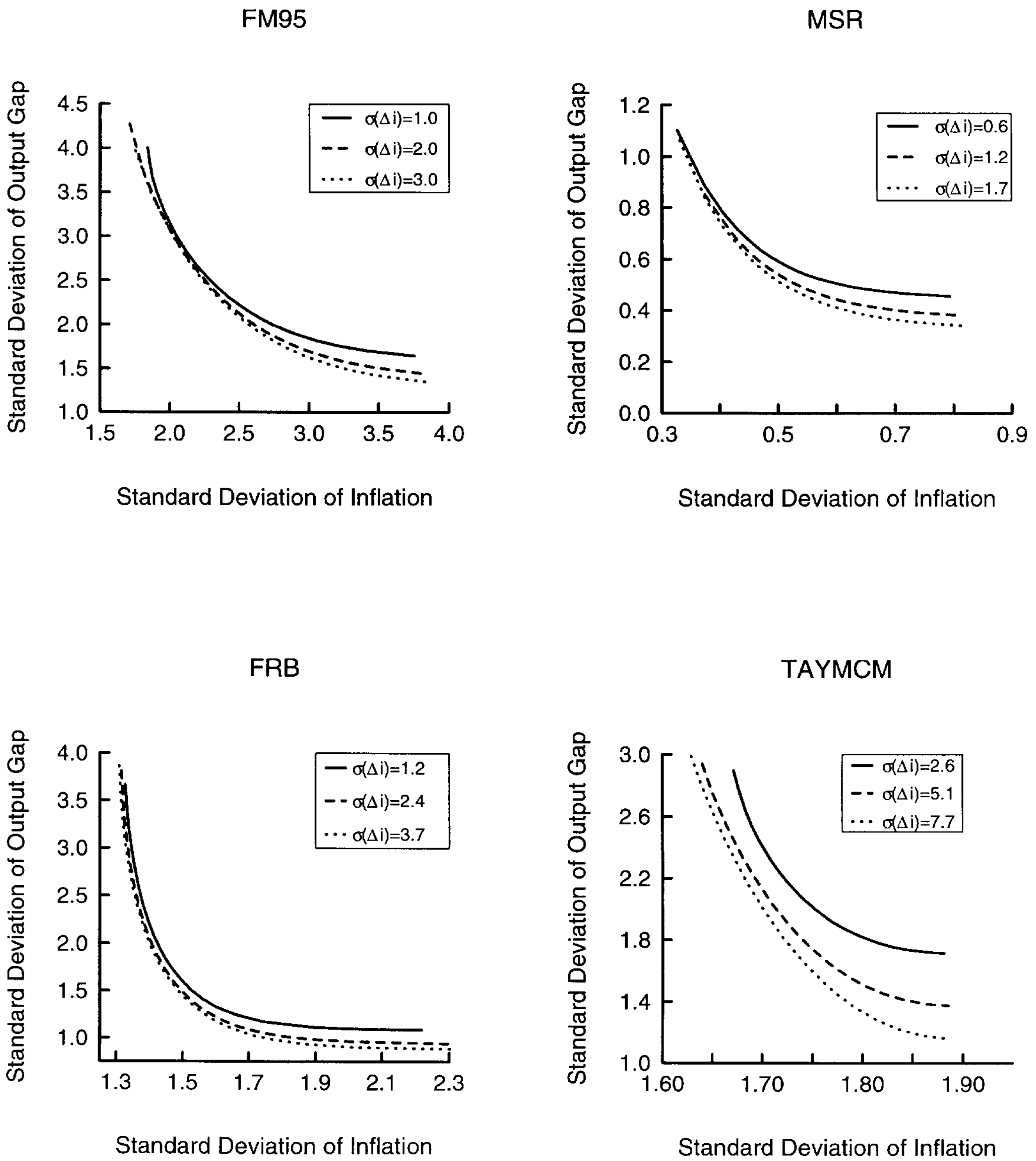


Figure 8: Aggregate Demand Shocks under Alternative Rules

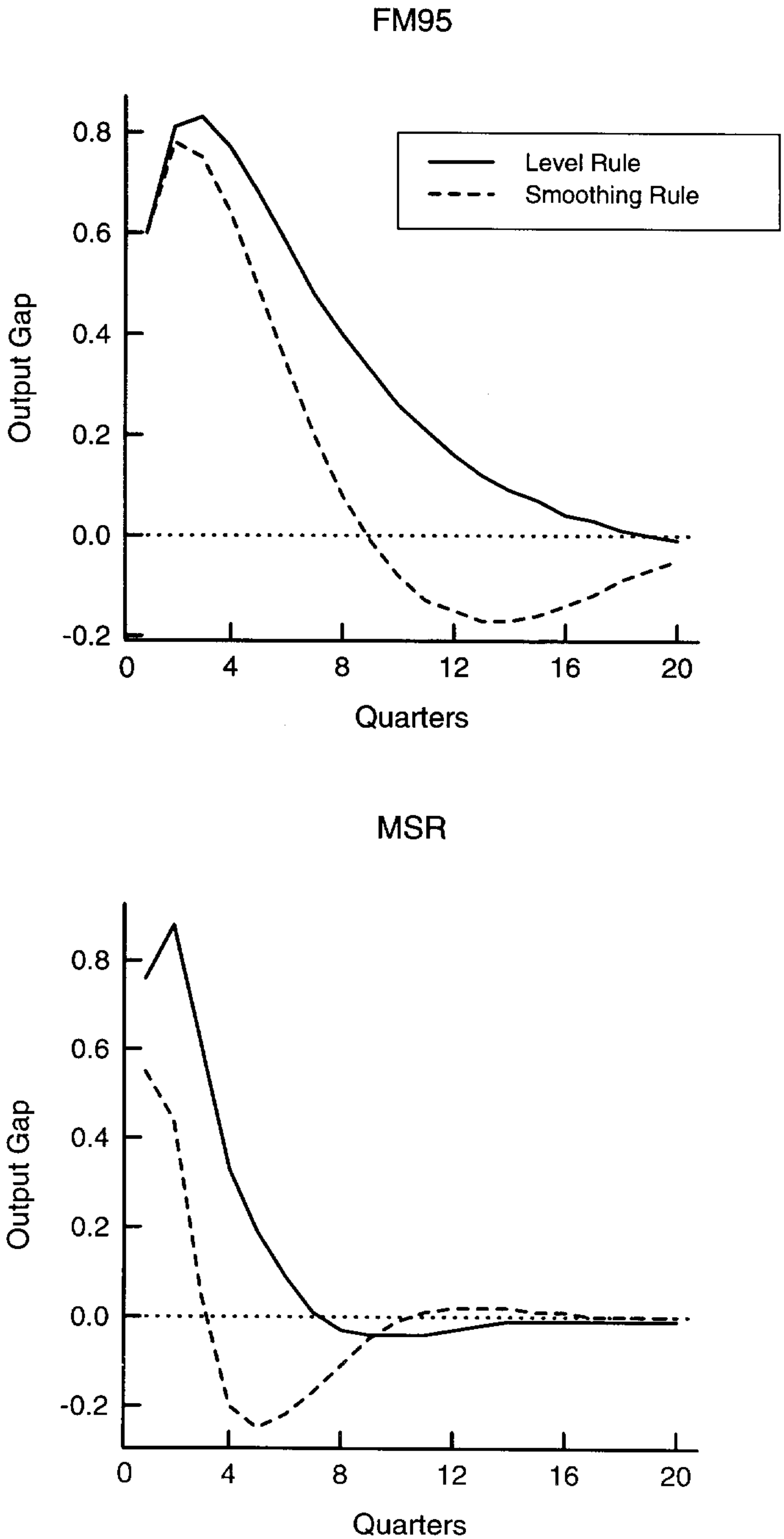


Figure 9: Performance of Simple Rules under Model Uncertainty

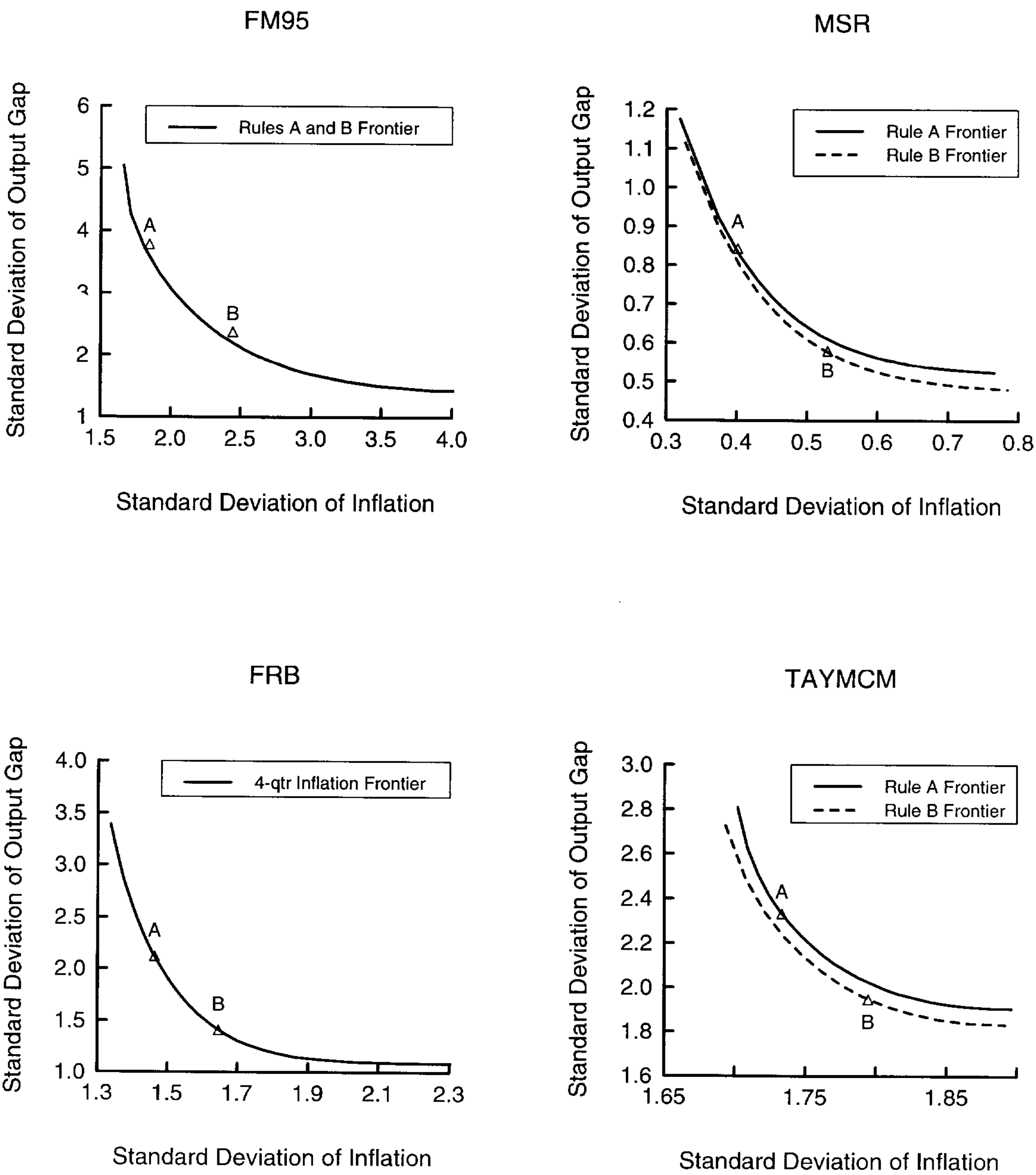


Figure 10: Two Complicated Rules from MSR