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# Capital Injection to Banks versus Debt Relief to Households

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# Capital Injection to Banks versus Debt Relief to Households*

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**Abstract**

I propose a dynamic stochastic general equilibrium model in which the leverage of borrowers as well as banks and housing finance play a crucial role in the model dynamics. The model is used to evaluate the relative effectiveness of a policy to inject capital into banks versus a policy to relieve households of mortgage debt. In normal times, when the economy is near the steady state and policy rates are set according to a Taylor-type rule, capital injections to banks are more effective in stimulating the economy in the long-run. However, in the middle of a housing debt crisis, when households are highly leveraged, the short-run output effects of the debt relief are more substantial. When the zero lower bound (ZLB) is additionally considered, the debt relief policy can be much more powerful in boosting the economy both in the short-run and in the long-run. Moreover, the output effects of the debt relief become increasingly larger, the longer the ZLB is binding.

***JEL Classification:*** E17, E44, E52, E62, G1, G21, H12

***Keywords:*** *capital injection to banks, debt relief to households, housing debt crisis, macro-financial linkages, leverage, zero lower bound.*

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# 1 Introduction

The Great Recession, which was the largest and longest economic downturn in the postwar era of the U.S.,[1] was triggered and intensified by the housing debt crisis known as the subprime mortgage crisis. Inflation-adjusted house prices reached a peak in the first quarter of 2006. At the same time, mortgage delinquency rates started to rise from a historically low level. House prices plummeted from the second quarter of 2007 onwards. Losses for financial institutions materialized,[2] and credit spreads began to rise. Alongside these financial developments, private consumption slowed down from the second quarter of 2007 and became very weak after the fourth quarter of 2007, the official beginning of the Great Recession. Nonresidential investment started to decline from the next quarter. Finally, after the collapse of Lehman Brothers in September 2008, real GDP in 2008Q4 fell drastically by 8.2 percent in annual terms.

It may not be surprising that a collapse in house prices can put the economy into a severe recession through the interaction between the real and financial sector. Housing finance has played a prominent role in advanced economies. According to recent empirical work by Jordá et al. (2014), bank loans backed by real estate consisted of roughly 60% of total bank lending in 2010, compared to around 30% in the 1950s.[3] They find that a rapid increase of home mortgages has mainly contributed to this substantial change in the lending business of banking. In addition, most of the banking crises in advanced economies were associated with boom-bust cycles in house prices. Reinhart and Rogoff (2009) show that five major banking crises during the second half of the twentieth century shared a common pattern[4]: a surge of house prices in the run-up to a crisis is followed by a sharp decline in the crisis year and in subsequent years together with a prolonged deep recession.

As one of the policy measures to mitigate the severity of the housing debt crisis and ensuing deep economic downturn, the U.S. government promptly used about \$500 billion, 3.4% of 2008 GDP, to support the U.S. financial sector. The government injected capital worth of \$245 billion into the U.S. banking sector through the Troubled Assets Relief Program (TARP).[5] It rescued American International Group (AIG), one of the world's major insurance companies, with \$67.8 billion of the TARP funds through Treasury purchase of AIG preferred equity, aside from Federal Reserve loans to AIG of maximum \$116.8 billion.[6] In addition to financial rescues through the

[1] According to the NBER's Business Cycle Dating Committee, the Great Recession started at 2007Q4 and ended at 2009Q2. During the period real GDP fell by 4.2 percent.

[2] In 2007Q2, New Century Financial Corporation, one leading subprime mortgage lender in the U.S., filed for bankruptcy. Concurrently, the charge-off rate on home mortgages for all U.S. commercial banks started to rise rapidly.

[3] Jordá et al. (2014) construct a long-run dataset on a wide range of private credit that includes credit to households and credit to firms by commercial banks as well as by other financial institutions such as saving banks, credit unions, and building societies.

[4] Spain (1977), Norway (1987), Finland (1991), Sweden (1991), and Japan (1992).

[5] The Emergency Economic Stabilization Act authorized the U.S. Treasury to purchase "troubled assets" worth of \$700 billion in October 2008. Part of the TARP funds, \$245 billion, were used to increase banks' capital. Most banks received funds through the Capital Purchase Program, while Bank of America and Citigroup additionally received \$20 billion each under the Targeted Investment Program.

[6] The Treasury purchased AIG preferred stock twice: the first purchase was \$40 billion in November 2008 and the

TARP, the housing government-sponsored enterprises, Fannie Mae and Freddie Mac, were nationalized one week before the Lehman Brothers' collapse. The U.S. government committed to putting up to $200 billion into each company and actually injected $187.5 billion capital into the two companies to cover their losses.

For households, the U.S. government pledged only $37.5 billion to refinance home mortgages of those who were in a negative equity position due to the sharp decline in house prices. This amount was tiny compared to the funds to support the financial sector. On top of that, less than one half of the pledged funds, $18 billion, have been actually spent. Table 1 reports this stark contrast between the two types of post-crisis government interventions.

Table 1: THE COMPARISON OF FINANCIAL SECTOR SUPPORT PROGRAMS AND HOUSEHOLD SUPPORT PROGRAMS DURING THE GREAT RECESSION

| | Amounts spent (billions) | % of 2008 GDP |
|---|---|---|
| Financial sector support programs | $500.2 | 3.4% |
| TARP: Capital Purchase Program etc. | $244.9 | 1.66% |
| TARP: AIG | $67.8 | 0.46% |
| Fannie Mae and Freddie Mac | $187.5 | 1.28% |
| TRAP: Mortgage refinance programs | $18.0 | 0.12% |

Source: Monthly TARP Update for 01/10/2015 and the Bailout Scorecard( projects.propublica.org/bailout)
Note: Mortgage refinance programs include Making Home Affordable Program ($12.2 billion), HFA Hardest-Hit Fund ($5.7 billion) and FHA Short Refinance ($0.02 billion).

Many prominent economists such as Mian and Sufi (2014), Stiglitz (2010), Shiller (2012), and Geanakoplos (2010) criticize this approach biased toward the rescue of financial institutions and argue that more grants for household debt reduction would have provided a significant boost to the economy lacking aggregate demand. Mian and Sufi (2014) claim that the biggest policy mistake of the Great Recession was not to push for mortgage write-downs more aggressively. In contrast, leading policy makers at that time, such as Geithner (2014), Summers (2014), and Bernanke (2015), defend the decisions to give priority to financial rescues and emphasize the importance of the credit channel working through financial institutions, even though they also admit that more active policies regarding household debt and foreclosure could have been beneficial.

In this paper, I propose a dynamic stochastic general equilibrium (DSGE) model in which the leverage of borrowers as well as banks and housing finance play a crucial role in model dynamics. The model is then used to evaluate the relative effectiveness of a policy to inject capital into the financial sector versus a policy to relieve households of mortgage debt.

The model combines several macro-financial linkages identified in the literature.[7] It includes two types of households, entrepreneurs and banks. Heterogeneity in the household sector is intro-

second was $27.8 billion in January 2009.

[7]Wieland et al. (2016) classify the recent development of macro-financial DSGE models into three categories: financial accelerator models for corporate investment financing, models with housing finance, and models with an explicit role of banks. I combine three different modelling approaches emphasizing a role of housing finance.

duced à la Iacoviello (2005): one household type is more patient than the other. In equilibrium, patient households become savers and ultimately supply funds to the economy, while impatient households are borrowers. The impatient households and entrepreneurs borrow funds from banks using real estate as collateral. My model deviates from the standard housing finance models in which borrowing is restricted to a certain fraction of collateral and there is no default. I introduce an agency problem between borrowers and banks by using the costly state verification (CSV) setup of Gale and Hellwig (1985). It implies that the model allows for default in equilibrium. Unlike Bernanke et al. (1999), I assume that the contractual interest rates are pre-determined rather than state-contingent. Accordingly, banks make zero expected profits in the perfectly competitive retail loan market, but ex-post profits mostly differ from zero. In my model, banks face a leverage constraint making the deviation of the leverage ratio from its target costly as in Gerali et al. (2010). With this leverage constraint, realized profits or losses can affect credit supply. The financial frictions described above are embedded into an otherwise standard New-Keynesian model with price and nominal wages rigidities.

The relationship between interest rate spreads and the related leverage ratios can describe key macro-financial linkages in the model. The risky debt contracts imply that the interest rate spread of each contractual loan rate over the wholesale loan rate, the rate that serves as a benchmark in the retail lending business, positively depends on the leverage of each borrower. Similarly, the bank's optimal decision shows that the interest rate spread of the wholesale loan rate over the deposit rate positively varies with the bank's leverage position. For example, when the leverage of impatient households decreases for some reason, the lending rate spread of home mortgages also narrows reflecting a decline in default. Faced with lower funding costs, borrower households increase consumption. Meanwhile, realized bank profits due to lower default costs help expanding credit availability. All other things being equal, it further boosts the expenditure of credit-constrained agents, impatient households and entrepreneurs.

Having in mind that most of the funds to support the financial sector presented in Table 1 were injected or committed in one or two quarters after the announcement, I model each policy as one-time transfer from credit-unconstrained(patient) households to either banks or credit-constrained(impatient) households in policy experiments. The capital injection to banks increases the current period's net worth of banks, while the debt relief to credit-constrained households reduces the outstanding home mortgages. The main findings from the policy experiments are the following:

When the economy is near the steady-state and policy rates are set according to a Taylor-type rule, the capital injection to banks is more effective in stimulating the economy over the long run. Even though the debt relief to credit-constrained households has a stronger effect on output for the first year, the capital injection policy dominates from the second year onward. The capital injections lead investment to increase, which in turn expands production capacity and results in lower inflation. On the contrary, the debt relief is inflationary and calls for an increase of the policy rate, which reduces investment and the consumption of credit-unconstrained households. In the middle of the housing debt crisis, however, a debt relief policy can be more effective. It is because in such a highly-leveraged situation, this policy can reduce the default risk posed by high leverage to a greater extent, thereby resulting in a lower lending rate spread of home mortgages, smaller wasteful foreclosure costs, and a greater short-run stimulus for consumption.

When in addition the zero lower bound (ZLB) constraint is considered, both policies give rise to larger effects on output and help the economy to escape from a liquidity trap earlier than without any policy.[8] More interestingly, the effects of the debt relief policy are magnified. The policy-induced inflation under the ZLB constraint leads to a lower real interest rate. The decrease in the real rate boosts investment as well as consumption, or at least significantly weakens crowding-out effects. Therefore, in this environment the debt relief can be much more effective in stimulating the economy both in the short-run and in the long-run. Moreover, the effects of the debt relief policy on output become increasingly larger as the number of periods that the policy rate is constrained at zero increases.

My model builds on a large literature incorporating financial frictions into a DSGE model, including the prominent groundwork such as Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke et al. (1999). This earlier work and most of the subsequent research focus on an agency problem between financial intermediaries and their borrowers. These kinds of financial frictions imply that the balance sheets of borrowers become a key factor to explain macro-financial linkages by affecting credit demand. Recently, in particular after the recent global financial crisis, there has been a growing literature focusing on the agency problem between financial intermediaries and their creditors (e.g. Gertler and Karadi (2011), Kiley and Sim (2014), and Christiano and Ikeda (2013)). In those approaches, the balance sheets of financial institutions play an important role in real economy by shifting credit supply. This paper contributes to another growing literature that considers financial frictions in both credit demand and credit supply at the same time and puts an emphasis on their interactions. It includes Gerali et al. (2010), Benes and Kumhof (2011), Iacoviello (2015), and Clerc et al. (2015).

This paper is also related to recent work analyzing the macroeconomic effects of housing prices during the Great Recession. Liu et al. (2013) and Guerrieri and Iacoviello (2015a) find that a collapse in housing prices can explain most of the sharp decline in aggregate demand, while Justiniano et al. (2015) argue that such fall in housing prices was not enough to put the economy into a deep recession. As for the modelling of home mortgage default I follow recent approaches that model default as a put option and impose no direct foreclosure costs on borrower households (for instance, Forlati and Lambertini (2011), Quint and Rabanal (2014), Landvoigt (2014), and Jeske et al. (2013)).

Regarding to policy experiments, a number of papers analyze the macroeconomic consequences of injecting more capital into banks using a DSGE model with a leverage constraint for banks (see Kollmann et al. (2013), Hirakata et al. (2013), van der Kwaak and van Wijnbergen

[8] A liquidity trap is usually defined as the situation in which policy rates cannot fall below zero given that hoarding cash offers an alternative to holding deposit.

(2014), Kiley and Sim (2014), Guerrieri et al. (2015) etc.). Most of them find that the capital injection policy has positive effects on output since it increases credit supply to the productive but credit-constrained sector. In contrast, only a few investigate the macroeconomic effects of reducing household debt. Guerrieri and Iacoviello (2015a) analyze the effects of a lump-sum transfer from credit-unconstrained households to credit-constrained households using a DSGE model with the presence of an occasionally binding constraint, and find that such a transfer can have sizable effects on output when the borrowing constraint binds. Mian and Sufi (2014) estimate the macroeconomic effects of the introduction of shared-responsibility mortgages, that in essence feature an automatic principal reduction when housing prices decline below the purchasing level.[9] They put into perspective several empirical studies such as Mian et al. (2015), Mian et al. (2013) and Nakamura and Steinsson (2014). They argue that the output effects of new mortgage contracts would be large enough to substantially reduce the severity of the recession. Dogra (2014) uses a simple model in which the economy hits the ZLB by household deleveraging and analyzes the effects of debt relief modeled as a targeted transfer. He finds that debt relief stimulates the economy, but the anticipation of debt relief leads to overborrowing. He shows, nevertheless, that optimal policy is still involved in the use of debt relief up to a certain level.

My contributions to the literature are, first, to design a rigorous policy experiment to reduce households' debt in a structural macro model and to compare the relative effectiveness of this policy and the policy to increase banking capital. Second, as an additional novelty, this paper evaluates those policies considering the zero lower bound, following a new literature to assess fiscal stimulus with the assumption of monetary accommodation (see Cogan et al. (2010), Eggertsson and Krugman (2012), and Coenen et al. (2012)). Lastly but not least, the model proposed in this paper applies the financial accelerator mechanism to business loan contracts collateralized by commercial real estate. With this modeling device the model implies that a decline in housing prices leads to a decrease in nonresidential investment as well as a rise in default on business loans, which is consistent with the empirical evidence. If a standard modelling setup for housing as in Iacoviello and Neri (2010) is simply combined with a standard risky debt contract for corporate financing as in Bernanke et al. (1999), then the model implies that a fall in housing prices accompanies a business investment boom.

The remainder of this paper is organized as follows. Section 2 describes the model economy and defines the competitive equilibrium. Section 3 explains the calibration strategy and its results. Section 4 analyzes the results from a series of simulations, and Section 5 concludes.

[9]Shared-responsibility mortgages have two important features that are distinct from the existing ones: the bank offers the protection to borrowers when house prices decline below the purchasing price, while the bank obtains 5% capital gains when house prices increase over the purchasing level.

## 2 The Model Economy

Time is discrete and quarterly. The economy is populated by a continuum of two types of infinitely-lived households. Each household has unit mass. They differ in their discount factors. One type is more patient than the other. A household obtains utility from consumption and housing services and disutility from labor. Within the household, perfect risk-sharing is provided to its members. The nominal wages are set by each type's monopolistic labor union. Each of the patient households has a large number of entrepreneurs, and owns banks, intermediate goods producing firms, retail firms, and capital goods producing firms.

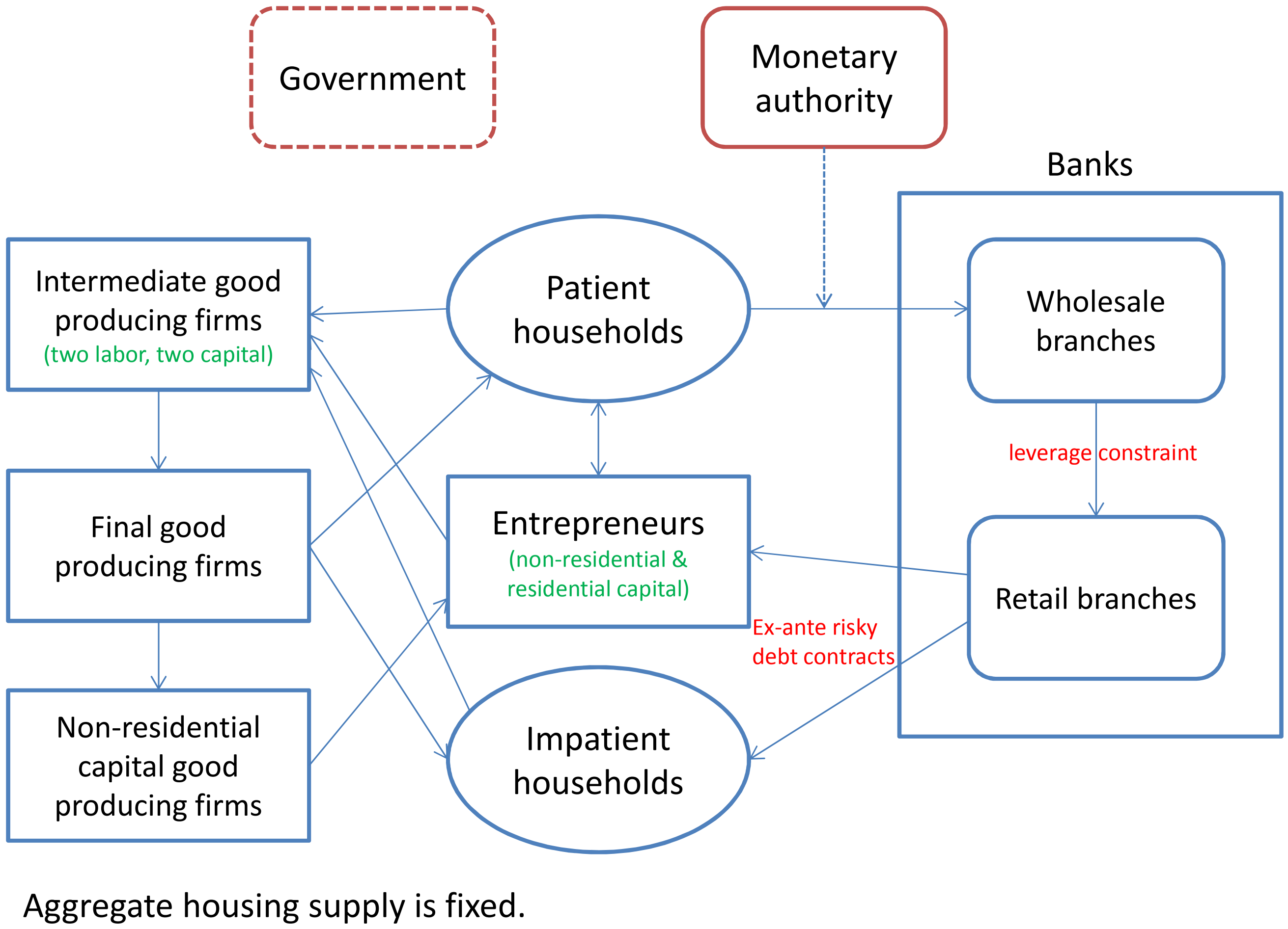
Banks channel funds from patient households (and its own net worth) to impatient households and entrepreneurs. Each bank consists of a wholesale branch and two retail branches. One retail branch deals with home mortgages, while the other handles business loans. The wholesale branch issues wholesale loans to the two retail branches subject to a leverage constraint such that it pays a pecuniary cost for the deviation of the bank net worth to asset ratio from its target. When it comes to the retail lending business, an agency problem arises because the return to the underlying assets posted as collateral is subject to idiosyncratic risk and the realization of an individual shock can only be observed by the bank after paying some cost. Consequently, each retail branch makes an ex-ante risky debt contract with its borrowers. Bank profits or losses are accumulated into its net worth after a fraction of the net worth is transferred to patient households.

Entrepreneurs combine loans with their net worth to purchase raw nonresidential capital and new residential capital. Then they provide the composite capital services to intermediate goods producing firms that use them with two types of labor to produce intermediate goods. Retail firms operate under monopolistic competition and are subject to implicit costs to adjust nominal prices following Calvo-style contracts. The constant-elasticity-of-substitution (CES) aggregates of these goods are converted into homogenous final goods. Capital goods producing firms purchase previously installed depreciated capital from entrepreneurs and investment goods from final good producing firms, and produce new installed capital subject to investment adjustment costs.

Aggregate housing supply is assumed to be fixed.[10] The central bank sets the nominal risk-free interest rate according to a Taylor-type rule. The government can collect lump-sum taxes from patient households and give them out to other agents. The structure of the model is depicted in Figure 1. In the following, I describe the decision problems of each agent and define the competitive equilibrium of the model economy.

[10]Basically, I exclude residential investment and its spillover effects on the broad economy from the analysis.

Figure 1: MODEL STRUCTURE



## 2.1 Households

The household sector is composed of two types of households. The discount factor of patient households is higher than that of impatient households ($\beta^P > \beta^I$). Each household ($s \in \{P, I\}$) maximizes the expected discounted sum of per-period utility:

$$V_H^s = E_0 \sum_{t=0}^{\infty} (\beta^s)^t \{\Gamma^s \log(C_t^s - \varepsilon C_{t-1}^s) + \chi_t^s \log H_t^s - \psi_l \frac{(L_t^s)^{1+\varphi}}{1+\varphi}\} \tag{1}$$

$C_t^s$ denotes consumption, $H_t^s$ refers to the housing stock owned by each household, $L_t^s$ denote labor supplied. I assume habit formation in consumption. As in Iacoviello (2005), utility from housing services is proportionate to the housing stock, and utility is separable in consumption and housing. $\Gamma^s$ is used for normalization such that the marginal utilities of consumption at the non-stochastic steady state are the inverse of consumption: $\Gamma^s = \frac{1-\varepsilon}{1-\beta^s\varepsilon}$. I allow for the possibility that each type of households values one unit of housing services differently. Housing preferences are subject to shocks. A decrease in $\chi_t^s$ moves preferences away from housing and towards consumption and leisure, so that housing demand decreases and, in the end, housing prices fall.[11] The shock process for each type is given by:

$$\log(\chi_t^s) = (1-\rho_\chi)\log(\chi^s) + \rho_\chi \log(\chi_{t-1}^s) + \varepsilon_{\chi,t} \quad s \in \{P, I\}. \tag{2}$$

In equilibrium, patient households are savers and impatient households are borrowers. For simplicity, I describe each type's decision problem taking these equilibrium outcomes into account.

### 2.1.1 Patient households

The budget constraint of patient households in real terms is:

$$C_t^P + Q_{h,t}(H_t^P - H_{t-1}^P) + D_t = \frac{w_t^P L_t^P}{x_{w,t}^P} + R_{t-1}D_{t-1} + \Upsilon_t^P. \tag{3}$$

$Q_{h,t}$ stands for housing prices, $D_t$ is a one-period deposit at period $t$, $R_{t-1}$ is the real gross interest rate on the last period's deposit $D_{t-1}$, $w_t^P$ is the real wage rate for patient workers, $x_{w,t}^P$ is the wage mark-up between the wage paid by intermediate goods producing firms and the wage paid to households, which accrues to the respective labor union. $\Upsilon_t^P$ stands for profits and net transfers. It includes profits from intermediate goods producers and the labor union, net transfers from banks and entrepreneurs, and lump-sum taxes to government. Patient households earn labor and non-labor income and spend it to consume, to purchase housing and to save. They maximize their life-time utility, Eq. (1), by choosing $\{C_t^P, H_t^P, D_t, L_t^P\}$ given the budget constraint, Eq. (3).

[11] For this reason, the shock on housing preferences is also called as a housing demand shock.

### 2.1.2 Impatient households

The budget constraint of impatient households in real terms is given by:

$$C_t^I + Q_{h,t}H_t^I + ac_{zh,t} - Z_t^I = \frac{w_t^I L_t^I}{x_{w,t}^I} + \int_0^\infty \max\{\omega_t Q_{h,t}H_{t-1}^I - R_{t-1}^L Z_{t-1}^I, 0\} dF(\omega_t) + \Upsilon_t^I. \quad (4)$$

$Z_t^I$ denotes a one-period mortgage loan at period $t$, $R_{t-1}^L$ is the real gross lending interest rate on last period's loan $Z_{t-1}^I$. $w_t^I$ is the real wage rate for impatient workers, $x_{w,t}^I$ is the wage mark-up that the labor union of impatient workers charges. $\Upsilon_t^I$ is profits from the labor union and net lump-sum transfer from government. The term $ac_{zh,t}$ refers to mortgage adjustment costs and its functional form is quadratic: $ac_{zh,t} = \frac{\kappa_{zh}}{2}\frac{(Z_t^I - Z_{t-1})^2}{Z^I}$. This reflects the fact that home mortgages are, in reality, highly long-term and involve many pecuniary and institutional impediments to quick loan adjustments.

I introduce the default risk in housing markets. The value of an individual house is subject to a unit-mean idiosyncratic shock $\omega_t$, which is drawn from the log-normal distribution: $\log(\omega_t) \sim \mathcal{N}(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2)$. The bank is assumed to offer non-recourse home mortgage loans to the individual members of a household, who use their individual housing as collateral. I further assume that there is no direct cost for households who default. Default, namely, is modeled as a put option. Each member indexed by $j$ decides whether or not to default based on the realization of an individual shock with the aim to maximize the individual net worth.

$$\max\{\underbrace{\omega_t Q_{h,t}H_{t-1}^I(j) - R_{t-1}^L Z_{t-1}^I(j)}_{\text{No default}}, \underbrace{0}_{\text{Default}}\}$$

The optimal decision rule puts the default threshold $\bar{\omega}_t$ at:

$$\bar{\omega}_t = \frac{R_{t-1}^L Z_{t-1}^I(j)}{Q_{h,t}H_{t-1}^I(j)} = \frac{R_{t-1}^L Z_{t-1}^I}{Q_{h,t}H_{t-1}^I} = \frac{m_{t-1}}{\Delta_{h,t}} \quad (5)$$

The individual members will default on mortgages if mortgage repayment obligations are greater than their housing values. As each member's holdings of mortgages are proportional to that of housing, the index $j$ can be dropped. $\bar{\omega}_t$ can be expressed in terms of the loan-to-value ratio of the previous period, $m_{t-1} = \frac{R_{t-1}^L Z_{t-1}^I}{Q_{h,t-1}H_{t-1}^I}$, and the ex-post average return on housing, $\Delta_{h,t} = \frac{Q_{h,t}}{Q_{h,t-1}}$. The default threshold $\bar{\omega}_t$ increases in the household's leverage and decreases in the realized return on housing. Using the default threshold $\bar{\omega}_t$, the budget constraint can be rewritten as:

$$C_t^I + Q_{h,t}H_t^I + ac_{zh,t} - Z_t^I \leq \frac{w_t^I L_t^I}{x_{w,t}^I} + [1 - \Gamma(\bar{\omega}_t)]Q_{h,t}H_{t-1}^I + \Upsilon_t^I. \quad (6)$$

where $\Gamma(\bar{\omega}_t)$ is the share of the housing value going to the bank.[12] Due to an agency problem,

[12] $F(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} dF(\omega_t^j; \sigma_\omega)$ is the foreclosure rate; $G(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} \omega_t^j dF(\omega_t^j; \sigma_\omega)$ denotes a fraction of the foreclosed houses of impatient households; $\Gamma(\bar{\omega}_t)$ are expressed by: $[1 - F(\bar{\omega}_t)]\bar{\omega}_t + G(\bar{\omega}_t)$.

the bank retail branch of home mortgages incurs a cost proportional to the value of foreclosed houses when it forecloses on mortgages. Accordingly, such a risky debt contract must satisfy the following ex-ante participation constraint of the bank:

$$E_t\{(\Gamma(\bar{\omega}_{t+1}) - \mu^B G(\bar{\omega}_{t+1}))Q_{h,t+1}H_t^I\} \geq R_t^r Z_t^I. \tag{7}$$

$\mu^B$ is a foreclosure cost parameter, $R_t^r$ is the wholesale real lending interest rate that serves as a benchmark rate in retail lending.[13] This constraint states that the expected gain of the bank's contribution to housing investment net of foreclosure cost is at least as high as its funding cost. Finally, impatient households maximize their life-time utility, Eq. (1), with respect to $\{C_t^I, H_t^I, Z_t^I, m_t, L_t^I\}$ subject to the budget constraint, Eq. (6) and the bank's participation constraint, Eq. (7).

It is worth noting that the participation constraint of the bank holds with equality. It means that the retail branch would make unexpected profits or losses in equilibrium depending on the realization of certain aggregate shocks. Such unexpected profits, $\varepsilon_{hb,t}$ are a function of endogenous variables, including the default threshold $\bar{\omega}_t$.

$$\varepsilon_{hb,t} = (\Gamma(\bar{\omega}_t) - \mu^B G(\bar{\omega}_t))Q_{h,t}H_{t-1}^I - R_{t-1}^r Z_{t-1}^I \tag{8}$$

In addition, the binding participation constraint of the bank implies that the interest rate spread of the contractual lending rate on mortgages $R_t^L$ over the wholesale lending rate $R_t^r$ is a function of the default threshold $\bar{\omega}_{t+1}$ in expectation.[14] As the right-hand side of Eq. (9) is an increasing function of $\bar{\omega}_{t+1}$, the interest rate spread rises when the leverage of borrowers increases.[15]

$$\frac{R_t^L}{R_t^r} = E_t\{\frac{\bar{\omega}_{t+1}}{\Gamma(\bar{\omega}_{t+1}) - \mu^B G(\bar{\omega}_{t+1})}\} \tag{9}$$

### 2.1.3 Nominal wage decisions

Nominal wage stickiness is introduced in a way analogous to nominal price stickiness as in Smets and Wouters (2007) and Iacoviello and Neri (2010). Each type of households supplies its homogenous labor services to the labor union that serves the interest of each type.[16] Each union differentiates labor services, sets nominal wages subject to Calvo-style adjustment frictions, and offers labor services to the respective labor packer. Each representative and competitive labor packer aggregates the differentiated labor services into the homogeneous labor services, which are hired by intermediate goods producing firms. The optimal wage rates set by each labor union

[13]Alternately, the wholesale lending rate can be thought as the rate at which banks would charge to notional zero-risk borrowers (see Benes and Kumhof (2011)).

[14]Plugging Eq. (5) into Eq. (7) holding with the equality, we obtain Eq. (9).

[15] Suppose that $\Omega(x) = \frac{x}{\Gamma(x) - \mu^B G(x)}$. Then $\Omega'(x) = \frac{(1-\mu^B)G(x) + x^2 f(x)}{(\Gamma(x) - \mu^B G(x)6)^2} > 0$ for all $x > 0$. Here, $f(x)$ is a density function of the log-normal distribution.

[16]The essence of wage staggering setting is to give workers bargaining power to decide wages for a certain period. Equivalently, we can assume that each monopolistic competitive household supplies differentiated labor services to the labor packer and sets nominal wages in a Calvo-style staggering contract.

together with the evolution formula for real wages imply the following wage Phillips curves:

$$\log(\frac{\Pi_{w^P,t}}{\Pi}) = \beta^P E_t \log(\frac{\Pi_{w^P,t+1}}{\Pi}) - \kappa_w^P \log(\frac{x_{w,t}^P}{x_w^P}) \tag{10}$$

$$\log(\frac{\Pi_{w^I,t}}{\Pi}) = \beta^I E_t \log(\frac{\Pi_{w^I,t+1}}{\Pi}) - \kappa_w^I \log(\frac{x_{w,t}^I}{x_w^I}) \tag{11}$$

where, $\Pi_{w^P,t} = w_t^P \Pi_t / w_{t-1}^P$ and $\Pi_{w^I,t} = w_t^I \Pi_t / w_{t-1}^I$ refer to wage inflation for patient and impatient households, respectively. $\kappa_w^P = (1-\theta_{w^P})(1-\beta^P\theta_{w^P})/\theta_{w^P}$, and $\kappa_w^I = (1-\theta_{w^I})(1-\beta^I\theta_{w^I})/\theta_{w^I}$ define the slope of each wage equation. $\Pi$ denotes the non-stochastic steady states of inflation. $x_w^P$ and $x_w^I$ are the wage markup of the patient and impatient households, each.

## 2.2 Entrepreneurs

Entrepreneurs are modeled in the same way as in Bernanke et al. (1999) and Christiano et al. (2014) with two exceptions. First, the contractual interest rates are predetermined rather than being state-contingent. Second, entrepreneurs deal with two types of capital, residential capital in addition to nonresidential capital.[17] Each patient household has a large number of entrepreneurs indexed by $j$, whose state is summarized by their net worth, $N_t^E(j)$. Each entrepreneur $j$ obtains a loan $Z_t^E(j)$ from the bank's retail branch for business loans, and combines it with his net worth $N_t^E(j)$ to purchase raw nonresidential capital $K_t(j)$ at a price of $Q_{k,t}$ and new residential capital $H_t^E(j) - H_{t-1}^E(j)$ at a price of $Q_{h,t}$. The balance sheet of each entrepreneur at the end of time $t$ is given by:

$$Q_{k,t}K_t(j) + Q_{h,t}H_t^E(j) = Z_t^E(j) + N_t^E(j). \tag{12}$$

At period $t+1$, entrepreneurs provide composite capital services, $\tilde{K}_t = H_t^E(j)^{\nu_k} K_t(j)^{1-\nu_k}$, to intermediate goods producers. The return to the composite capital, $\omega_{e,t+1}(R_{t+1}^k Q_{k,t}K_t(j) + R_{t+1}^h Q_{h,t}H_t^E(j))$, is assumed to be sensitive to both idiosyncratic and aggregate shocks. An idiosyncratic shock $\omega_{e,t}$ is modeled to follow a log-normal distribution: $\log\omega_{e,t} \sim N(-\frac{\sigma_{e\omega}^2}{2}, \sigma_{e\omega}^2)$, with $E_t\omega_{e,t} = 1$. The rates of return to nonresidential and residential capital are given by:

$$R_{t+1}^k = \frac{r_{k,t+1} + (1-\delta_k)Q_{k,t+1}}{Q_{k,t}}, \tag{13}$$

$$R_{t+1}^h = \frac{r_{h,t+1} + Q_{h,t+1}}{Q_{h,t}}. \tag{14}$$

$r_{k,t+1}$ and $r_{h,t+1}$ are competitive market rental rates for nonresidential and residential capital, respectively. Nonresidential capital depreciates at a quarterly rate of $\delta_k$. As can be seen in Eqs. (13) and (14), an aggregate shock can affect the return to composite capital via either the rental rates or capital prices or both. Note that $R_{t+1}^k$ and $R_{t+1}^k$ are equal across entrepreneurs indexed by $j$. Similarly to the default threshold of impatient households, Eq. (5), the decision rule for the

[17] In this paper, residential capital is used interchangeably with commercial real estate.

entrepreneurs' default threshold is expressed by:

$$\bar{\omega}_{e,t+1} = \frac{R_t^E Z_t^E(j)}{R_{t+1}^k Q_{k,t} K_t(j) + R_{t+1}^h Q_{h,t} H_t^E(j)}. \tag{15}$$

$R_t^E$ denotes the real gross lending rate on the business loan $Z_t^E$. It is worth noting that $\bar{\omega}_{e,t+1}$ is independent of the entrepreneur's net worth and thus her net worth only matters for the size of certain project. In what follows, I drop the index, $j$, for notational conveniences. If the realized idiosyncratic shock is below the default threshold, $\omega_{e,t+1} < \bar{\omega}_{e,t+1}$, then entrepreneurs default and hand over all remaining resources to the bank. Meanwhile, the business loan branch has to pay an auditing cost $\mu^E$ proportional to the assets of the bankrupt entrepreneurs due to asymmetric information. If $\omega_{e,t+1} \geq \bar{\omega}_{e,t+1}$, entrepreneurs repay debt $R_t^E Z_t^E$ while keeping the surplus return to investment. Therefore, the contractual terms of such a risky debt must satisfy the following ex-ante participation constraint of the bank.

$$E_t\{(1-F(\bar{\omega}_{e,t+1}))R_t^E Z_t^E + (1-\mu^E)\int_0^{\bar{\omega}_{e,t+1}} \omega_{e,t}(R_{t+1}^k Q_{k,t} K_t + R_{t+1}^h Q_{h,t} H_t^E) f(\omega_{e,t}) d\omega_{e,t}\} \geq R_t^r Z_t^E. \tag{16}$$

Here, $F(\bar{\omega}_{e,t+1})$ is the default rate for business loans.[18] Finally, each entrepreneur maximizes his share of the expected gross return of capital investment in period $t+1$, $E_t\{(1-\Gamma(\bar{\omega}_{e,t+1})(R_{t+1}^k Q_{k,t} K_t + R_{t+1}^h Q_{h,t} H_t^E)\}$, subject to Eqs. (12) and (16). I define two variables for entrepreneurial leverage: $\phi_t^k = \frac{Q_{k,t} K_t}{N_t^E}$, $\phi_t^h = \frac{Q_{h,t} H_t^E}{N_t^E}$. Both leverage ratios increase when the entrepreneur's net worth decreases. Each leverage variable also increases when the respective asset prices decline. After some algebra, the entrepreneur's maximization problem can be reformulated as:

$$\max_{\phi_t^k, \phi_t^h, \bar{\omega}_{e,t+1}} \quad E_t\{(1-\Gamma(\bar{\omega}_{e,t+1}))(R_{t+1}^k \phi_t^k + R_{t+1}^h \phi_t^h)\} N_t^E \tag{17}$$

subject to

$$E_t\{[\Gamma(\bar{\omega}_{e,t+1}) - \mu^E G(\bar{\omega}_{e,t+1})](R_{t+1}^k \phi_t^k + R_{t+1}^h \phi_t^h)\} = R_t^r(\phi_t^k + \phi_t^h - 1)$$

The default threshold can be rewritten in terms of leverage variables: $\bar{\omega}_{e,t+1} = \frac{R_t^E(\phi_t^k + \phi_t^h - 1)}{R_{t+1}^k \phi_t^k + R_{t+1}^h \phi_t^h}$. As the first order conditions (see Eqs. (C.20) and (C.21)) imply that $E_t \frac{R_{t+1}^k}{R_t^r} = E_t \frac{R_{t+1}^h}{R_t^r}$, we find that the expectation of $\bar{\omega}_{e,t+1}$ increases with total leverage of entrepreneurs, $\phi_t^k + \phi_t^h$.[19] The retail branch for business loans would make unexpected profits in equilibrium according to the realization of a certain aggregate shock. Such profit surprises, $\varepsilon_{eb,t}$ are expressed by:

[18] In addition, $\Gamma(\bar{\omega}_{e,t+1})$ denotes a fraction of gross return of capital investment going to the banks, and $G(\bar{\omega}_{e,t+1})$ refers to a fraction of the defaulted value of composite capital. The mathematical expressions are same as in footnote 12.

[19] Under $E_t \frac{R_{t+1}^k}{R_t^r} = E_t \frac{R_{t+1}^h}{R_t^r}$, $E\{\bar{\omega}_{e,t+1}\} = 1 - \frac{1}{\phi_t^k + \phi_t^h}$.

$$\varepsilon_{eb,t} = (\Gamma(\bar{\omega}_{e,t}) - \mu^E G(\bar{\omega}_{e,t}))(R_t^k Q_{k,t-1} K_{t-1} + R_t^h Q_{h,t-1} H_{t-1}^E) - R_{t-1}^r Z_{t-1}^E \quad (18)$$

Also, reformulations of the participation constraint of banks using the default threshold $\bar{\omega}_{e,t+1}$ shows that the lending rate spread of business loans is a function of the default threshold $\bar{\omega}_{e,t+1}$ in expectation. As the right-hand side of Eq. (19) is an increasing function of $\bar{\omega}_{e,t+1}$, the lending rate spread increases when the entrepreneurial leverage goes up.[20]

$$\frac{R_t^E}{R_t^r} = E_t\{\frac{\bar{\omega}_{e,t+1}}{(\Gamma(\bar{\omega}_{e,t+1}) - \mu^E G(\bar{\omega}_{e,t+1}))}\} \quad (19)$$

At the end of the period $t+1$, a fraction $(1-\gamma)$ of each entrepreneur's net worth is transferred to his own household and each entrepreneur receives a lump-sum transfer $W^e$ from the household. Since the ex-post net worth of an individual entrepreneur is linear, we can simply integrate individual entrepreneurs' net worth and derive the evolution of total net worth of entrepreneurs as follows:

$$N_t^E = \gamma[(1 - \Gamma(\bar{\omega}_{e,t}))(R_t^k \phi_{t-1}^k + R_t^h \phi_{t-1}^h)\} N_{t-1}^E] + W^e \quad (20)$$

## 2.3 Banks

Following Gerali et al. (2010), I assume that there is a unit mass of banks and each bank consists of two retail branches and one wholesale branch. The first retail branch is responsible for providing home mortgages $Z_t^I$ to impatient households. The second retail branch gives out business loans $Z_t^E$ to entrepreneurs. I also assume that both retail branches operate in perfect competition. They obtain wholesale loans at the wholesale rate $R_t^r$ and sell them to final borrowers on the condition that the ex-ante participation constraint of the bank holds. Therefore, each retail branch sets the contractual lending rate according to Eqs. (9) and (19), respectively. Then the retail branches pass over resulting profits or losses to the wholesale branch.

The wholesale branch has a net worth $N_t$, which is accumulated out of retained profits, and collects deposits $D_t$ from patient households at the deposit rate $R_t$. With these funds the wholesale branch issues wholesale loans $W_t$ at the wholesale rate $R_t^r$ while paying a linear operating cost $\kappa_w W_t$ and a quadratic penalty cost when the bank net worth to asset ratio $N_t/W_t$ deviates from its target. The problem of the wholesale branch is to choose loans and deposits to maximize the discounted sum of expected profits subject to the balance-sheet constraint:

[20] Refer to the footnote 15.

$$\max_{\{W_t,D_t\}} \quad E_0 \sum_{t=0}^{\infty} (\beta^P)^{t+1} \Lambda^P_{0,t+1} \Pi_{wb,t+1} \tag{21}$$

$$\text{where,} \quad \Pi_{wb,t+1} = R^r_t W_t + \Pi_{hb,t+1} + \Pi_{eb,t+1} - R_t D_t - N_t - \kappa_w W_t - \frac{\phi_n}{2}(\frac{N_t}{W_t} - \nu_b)^2 N_t$$

subject to

$$W_t = Z^I_t + Z^E_t = N_t + D_t.$$

Here, $\Pi_{hb,t+1}$ denotes total profits of the home mortgage branch; $\Pi_{eb,t+1}$ refers to those of the business loan branch. Due to the binding participation constraints of the bank, profits or losses resulting from an ex-ante risky debt contract, $\varepsilon_{hb,t}$ and $\varepsilon_{eb,t}$ are an unexpected surprise by construction.[21] To improve empirical validity, I assume that $\varepsilon_{hb,t}$ and $\varepsilon_{eb,t}$ have persistent effects on the bank's net worth with a decaying factor ($\rho_h$ and $\rho_e$, respectively), albeit ad hoc.[22] Then $\Pi_{hb,t}$ and $\Pi_{eb,t}$ are expressed by:

$$\begin{aligned} \Pi_{hb,t} &= \varepsilon_{hb,t} + \rho_h \varepsilon_{hb,t-1} + \rho_h^2 \varepsilon_{hb,t-2} + \rho_h^3 \varepsilon_{hb,t-3} + \cdots, \\ \Pi_{eb,t} &= \varepsilon_{eb,t} + \rho_e \varepsilon_{eb,t-1} + \rho_e^2 \varepsilon_{eb,t-2} + \rho_e^3 \varepsilon_{eb,t-3} + \cdots. \end{aligned}$$

The above equations can be reformulated in autoregressive form.

$$\Pi_{hb,t} = \varepsilon_{hb,t} + \rho_h \Pi_{hb,t-1}, \tag{22}$$

$$\Pi_{eb,t} = \varepsilon_{eb,t} + \rho_e \Pi_{eb,t-1}. \tag{23}$$

The first order condition of the wholesale branch's problem is given by:

$$R^r_t - R_t = \kappa_w - \phi_n(\frac{N_t}{W_t} - \nu_b)(\frac{N_t}{W_t})^2. \tag{24}$$

Since the right-hand side of Eq. (24) is decreasing in the bank net worth to asset ratio around the steady state, the interest rate spread between the wholesale loan rate and the deposit rate increases (up to the first order approximation) when the bank's leverage goes up. All bank profits are reinvested in banking activity and a fraction ($\delta_b$) of the pre-profit bank net worth is transferred to its own household. Aggregate bank capital evolves according to

$$N_t = (1 - \delta_b) N_{t-1} + \Pi_{wb,t} \tag{25}$$

[21] $E_t[\varepsilon_{hb,t+1}] = 0$, $E_t[\varepsilon_{eb,t+1}] = 0$.

[22] I take an approach similar to Guerrieri et al. (2015) that employ an autoregressive process of order 1 for bank losses in their simulations based on an empirically-relevant scenario used in the stress tests for the U.S. banking sector. The introduction of the ad-hoc adjustment implies a stronger role of the bank's net worth in model dynamics than otherwise. It does not, however, change simulation results qualitatively.

## 2.4 Production sector and nominal rigidities

In order to introduce price rigidities, I differentiate between competitive intermediate goods producing firms and retail firms. Intermediate goods producing firms hire composite capital services from entrepreneurs and two types of labor from households and solve the following maximization problem given their production technology.

$$\max_{\{L_t^P, L_t^I, K_{t-1}, H_{t-1}^E\}} \{\frac{Y_t}{x_{p,t}} - w_t^P L_t^P - w_t^I L_t^I - r_{k,t} K_{t-1} - r_{h,t} H_{t-1}^E\} \tag{26}$$

$$\text{where,} \quad Y_t = [{H_{t-1}^E}^{\nu_k} {K_{t-1}}^{1-\nu_k}]^{\alpha} [{L_t^P}^{\nu_l} {L_t^I}^{(1-\nu_l)}]^{(1-\alpha)}.$$

$x_{p,t}$ is the price markup of final goods over intermediate goods. Retail firms operate in a regime of monopolistic competition and face Calvo-type nominal price frictions. Retailers buy intermediate goods $Y_t$ at the price $P_t^w$ in a competitive market, differentiate the goods and sell them at price $P_t$, which includes a markup $x_{p,t} = P_t/P_t^w$ over the marginal cost $P_t^w$. The CES aggregates of these goods are converted into homogenous final goods, which are purchased by households and capital good producing firms. In every period, each retail firm sets optimal prices with probability $1-\theta_\pi$ or indexes prices to the steady state inflation $\Pi$ with probability $\theta_\pi$, regardless of the history of its price adjustments. These assumptions deliver the following price Phillips curve:

$$\log(\frac{\Pi_t}{\Pi}) = \beta^P E_t \log(\frac{\Pi_{t+1}}{\Pi}) - \kappa_\pi \log(\frac{x_{p,t}}{x_p}) \tag{27}$$

where, $\kappa_\pi = \frac{(1-\theta_\pi)(1-\beta^P\theta_\pi)}{\theta_\pi}$ determines the sensitivity of inflation to changes in the price markup $x_{p,t}$ relative to its steady state value $x_p$.

Capital goods producing firms purchase previously installed depreciated capital from entrepreneurs and investment goods $I_t$ from final goods producing firms, and produce new installed capital subject to investment adjustment costs. The capital goods producer solves:

$$\max_{I_\tau} E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \{Q_{k,\tau} I_\tau - [1 + s_k(\frac{I_\tau}{I_{\tau-1}})] I_\tau\}, \tag{28}$$

where, $s_k(x) = \frac{\kappa_k}{2}(x-1)^2$. The aggregate nonresidential capital evolve according to:

$$K_{t+1} = (1-\delta_k) K_t + I_t. \tag{29}$$

## 2.5 Central Bank and Government

The central bank sets the risk-free nominal interest rate based on an interest rate feedback rule that allows for interest rate smoothing and reacts to annual inflation and output.

$$R_t^n = \max\{1, (R_{t-1}^n)^{\gamma_R} (\frac{\Pi_t^A}{\Pi^A})^{(1-\gamma_R)\gamma_\pi} (\frac{Y_t}{Y})^{(1-\gamma_R)\gamma_y} (R^n)^{1-\gamma_R}\}, \tag{30}$$

where, $\Pi_t^A$ is year-on-year inflation, which is expressed in quarterly terms as $\Pi_t^A = (P_t/P_{t-4})^{1/4}$.

The link between nominal and real interest rates is given by the Fisher equation: $R_t^n = R_t \cdot E_t \Pi_{t+1}$. Government uses fiscal policy only for redistributive purposes: $Tr_t^P + Tr_t^I = 0$. In the baseline model I ignore the redistributive role of the government.

## 2.6 Competitive Equilibrium

The model is closed by market clearing conditions. As aggregate housing supply is normalized to one, housing market clears as below:

$$H_t^P + H_t^I + H_t^E = 1 \tag{31}$$

By Walras' law, the good's market clears:

$$Y_t = C_t^P + C_t^I + [1 + s_k(\frac{I_t}{I_{t-1}})]I_t + Adj_t,$$

where, $Adj_t = \mu^B G(\bar{\omega}_t;) Q_{h,t} H_{t-1}^I + \mu^E G(\bar{\omega}_{e,t})(R_t^k Q_{k,t-1} K_{t-1} + R_t^h Q_{h,t-1} H_{t-1}^E) + \kappa_w W_t + \frac{\phi_n}{2}(\frac{N_t}{W_t} - \nu_b)^2 N_t + ac_{zh,t}$.

The **competitive equilibrium** consists of a set of prices $\{R_t, R_t^L, R_t^E, R_t^r, R_t^k, R_t^h, Q_{h,t}, Q_{k,t}, r_{h,t}, r_{k,t}, w_t^P, w_t^I, \Pi_t, x_{p,t}, x_{w,t}^P, x_{w,t}^I\}_{t=0}^{\infty}$ and a set of real allocations $\{C_t^P, C_t^I, L_t^P, L_t^I, \Lambda_t^P, \Lambda_t^I, H_t^P, H_t^I, H_t^E, D_t, Z_t^I, Z_t^E, K_t, N_t^E, \phi_t^k, \phi_t^h, \bar{\omega}_t, \bar{\omega}_{e,t}, N_t, \Pi_{hb,t}, \Pi_{eb,t}, I_t, Y_t\}_{t=0}^{\infty}$ for a given government policy $\{R_t^n, Tr_t^P, Tr_t^I\}_{t=0}^{\infty}$, a realization of exogenous variables $\{\varepsilon_{\chi,t}\}_{t=0}^{\infty}$ and initial conditions $\{H_{-1}^P, H_{-1}^I, H_{-1}^E, D_{-1}, Z_{-1}^I, Z_{-1}^E, K_{-1}, N_{-1}^E, N_{-1}\}$ such that

1. households of both types maximize the life-time utility given the prices;
2. each labor union maximizes its profits given the prices;
3. the entrepreneurs' allocations solve the problem (17) given the prices;
4. the banks' allocations solve the problem (21) given the prices;
5. the intermediate goods firms solve the problem (26) given prices;
6. the retail firms maximize their profits given the prices;
7. the capital goods producing firms solve the problem (28) given the prices;
8. the government budget constraint holds; and
9. markets clear.

A set of equations describing the equilibrium of the model is summarized in Appendix C.

## 3 Calibration

I divide the model parameters into two sets. For the first set of parameters I choose values mostly from the relevant literature. The second set of parameters is endogenously determined in the model to ensure that the model's steady-state is consistent with the empirical features related to macro aggregates as well as debt and real estate owned by households and nonfinancial businesses for the U.S. economy during 1991-2006.

### 3.1 Exogenously chosen parameters

Table 2: EXOGENOUSLY CHOSEN PARAMETERS

| Description | Parameter | Value |
|---|---|---|
| *(Households, Production sector, and Nominal rigidities)* | | |
| Discount factor, patient households | $\beta^P$ | 0.995 |
| Habit formation in consumption | $\varepsilon$ | 0.65 |
| Inverse of the Frisch elasticity | $\varphi$ | 1.0 |
| Disutility weight on labor | $\psi_l$ | 1.0 |
| Steady-state price markup | $x_p$ | 1.2 |
| Steady-state wage markup | $x_w$ | 1.2 |
| Probability of keeping prices fixed | $\theta$ | 0.85 |
| Probability of keeping wages fixed | $\theta_w$ | 0.9 |
| Investment adjustment cost | $\kappa_k$ | 2.0 |
| Steady-state inflation | $\Pi$ | 1.005 (2 %) |
| Wage share of impatient households | $1-\nu_l$ | 0.35 |
| *(Financial frictions)* | | |
| Foreclosure cost | $\mu^B$ | 0.17 |
| Monitoring cost | $\mu^E$ | 0.215 |
| Fraction of entrepreneurial net worth transferred to households | $1-\gamma$ | 1-0.982 |
| Target capital-to-asset ratio | $\nu_b$ | 0.1 |
| Wholesale funds operating cost | $\kappa_w$ | 0.015/4 |
| Bank capital adjustment cost | $\phi_n$ | 25 |
| Mortgage adjustment cost | $\kappa_{zh}$ | 4.0 |
| Autocorrelation of retail profits, loans to households | $\rho_h$ | 0.9 |
| Autocorrelation of retail profits, loans to entrepreneurs | $\rho_e$ | 0.9 |
| *(Others)* | | |
| Policy smoothing coefficient | $\rho_R$ | 0.7 |
| Policy reaction coefficient on inflation | $\rho_\pi$ | 2.0 |
| Policy reaction coefficient on output | $\rho_y$ | 0.5/4 |
| AR(1) coefficient on housing preference shock | $\rho_\chi$ | 0.97 |

Exogenously chosen parameters are presented in Table 2. The discount factor of patient households $\beta^P$ is set at 0.995, implying that the steady-state risk-free real interest rate is 2 percent. As for other conventional parameters, I choose standard values in the New Keynesian literature. These include the consumption habit parameter $\varepsilon$, the inverse Frisch elasticity of labor supply $\varphi$, the steady-state price markup $x_p$, the steady-state wage markup $x_w$, and the investment adjustment cost parameter $\kappa_k$. The disutility weight on labor steady $\psi_l$ is normalized to one. I set parameters governing the price and wage rigidities, $\theta$ and $\theta_w$, to 0.85 and 0.9, respectively. I choose values from the mid-upper range found in the literature in order to partially compensate the absence of several real or nominal frictions such as variable capital utilization and partial indexation of prices and wages to past inflation. The steady-state annual inflation is set to 2%. I set the steady-state wage share of impatient households to 0.35, which is in mid-range of the existing literature.[23] The resulting income share of these households in aggregate income is 24%.

In choosing the parameters related to financial frictions, I rely on a number of previous studies. As in Chatterjee and Eyigungor (2015), the foreclosure cost parameter $\mu^B$ is set to 0.17, which means that banks lose 17 percent of the values of houses in case of foreclosure. The parameter for the monitoring cost of business loans $\mu^E$ is taken from Christiano et al. (2014), who estimate it together with other structural parameters. The parameter governing the transfer from entrepreneurs to their respective households, $1-\gamma$, is determined to be 1-0.982. This lies in between the 1-0.973 value used by Bernanke et al. (1999) and the 1-0.985 by Christiano et al. (2014).

Due to the binding participation constraint of the bank, the model-implied profits and losses are unexpected and serially uncorrelated. This implication conflicts with the data that show persistence. To improve the fit of the model, I introduce AR(1) processes (Eqs. (22) and (23)). I set the autocorrelation coefficients, $\rho_h$ and $\rho_e$, at 0.9, following Guerrieri et al. (2015) who conduct comparative exercises with regard to an exogenous shock to bank capital losses with five structural macro-financial models.

The parameter for the bank capital adjustment cost $\phi_n$ is set to 25, which is roughly twice the estimated value of 11.5 in Gerali et al. (2010). It implies that the elasticity of the bank's interest rate spread with respect to the bank net worth is approximately twice as big as that in Gerali et al. (2010).[24] The targeted capital-to-asset ratio $\nu_b$ is set to 10 percent. I consider not only the regulatory minimum capital requirement of 8% but also the additional requirement implicitly enforced by market discipline, which seems to be consistent with U.S. data.[25] The operating cost parameter at the wholesale branch of banks, $\kappa_w$, is put to 0.015/4, implying that the steady-state interest rate spread of the wholesale loan rate over the deposit rate is 1.5% in annual terms. The parameter governing mortgage adjustment costs is set to 4. Since there is no prior information, I conduct the sensitivity analysis in Appendix B.

As for the interest rate rule's specification, I use 0.7 for interest rate smoothing, 2.0 for the

[23] The wage share of credit-constrained households used in other studies is as follows: 0.36 in Iacoviello (2005), 0.21 in Iacoviello and Neri (2010), 0.42 in Guerrieri and Iacoviello (2015a), etc. Using the Survey of Consumer Finances (SCF), Kaplan and Violante (2014) estimate that between 17.5% and 35% of U.S. households are liquidity-constrained.

[24] Refer to Eq. (32) in footnote 36.

[25] The average of the equity to asset ratios for the U.S. commercial banks during 2004-2006 is 9.9%.

reaction coefficient on inflation, and 0.5/4 for the reaction coefficient on output. Lastly, I employ a fairly persistent AR(1) process for the housing preference shock as in Iacoviello and Neri (2010) and Liu et al. (2013).

### 3.2 Endogenously determined parameters

Endogenously determined parameters are reported in Table 3 together with empirical targets used for calibration. The steady-state default threshold for home mortgages $\bar{\omega}$ and the standard deviation of an idiosyncratic shock in households' housing $\sigma_{\omega}$ are chosen such that the loan to value ratio of impatient households is 85% and the annual foreclosure rate for home mortgages is 1.5%. The former is in line with the average loan to value ratios of first-time home buyers for the 1990s and early 2000s estimated by Duca et al. (2011), and the latter is the average foreclosure rate for 1991-2006 reported by the Mortgage Bankers Association. In the similar way, the steady-state default threshold for business loans $\bar{\omega}_e$ and the standard deviation of an individual shock in entrepreneurs' capital services $\sigma_{\omega_e}$ are calibrated to agree both the average of debt to net worth ratio of nonfinancial business for 1991-2006, 0.42, and the annual business default rate estimated by Christiano et al. (2014), 2.25%.[26]

The capital share in production $\alpha$, the commercial real estate share in production $1-\nu_k$, and the depreciation rate $\delta_k$ are jointly calibrated to match three sample averages for 1991-2006: the nonresidential fixed assets to output ratio of 6.5, the share of business residential fixed assets in total residential fixed assets of 11%, and the nonresidential investment to output ratio of 14.6%.[27] The resulting $\alpha$, $1-\nu_k$, and $\delta_k$ are equal to 0.307, 0.050, and 0.0226, respectively.

The parameters governing the steady-state housing weight in utility for the patient and impatient households, $\chi^P$ and $\chi^I$, are jointly determined such that the home real estate to annual output ratio is 1.7 and the share of home mortgages in total loans, home mortgages plus business loans, is 45%.[28] The resulting $\chi^I$ is 0.190, which is approximately six times as big as the calibrated $\chi^P$ of 0.032. These calibration results imply that the impatient households, as a whole, would exert more influence on housing prices than the case when $\chi^P = \chi^I$ is assumed. Guerrieri and Iacoviello (2015a) find that in their housing finance model with the assumption of $\chi^P = \chi^I$, housing services are primarily priced by patient households. Geanakoplos (2010) argues, however, that houses

[26] Debt of nonfinancial business is the sum of the credit market instruments of both nonfinancial corporate business and nonfinancial noncorporate business. Net worth is the sum of net worth of nonfinancial corporate business at market prices and net worth of nonfinancial noncorporate business. All the data are taken from Tables B.102 and B.103 in Flow of Funds Accounts.

[27] To arrive at these numerical values, I first construct output series, which is consistent with the model definition - the sum of consumption and nonresidential investment, using the chain-aggregation methods outlined in Whelan (2002). Then I compute the nonresidential fixed assets to output ratio and the nonresidential investment to output ratio by dividing the respective variables by the model-consistent output. Business residential fixed assets are defined as the sum of ones owned by corporate business and noncorporate business, such as sole proprietorships and partnerships. I compute its share in all residential fixed assets. I use National Economic Accounts (Tables 1.1.3, 1.1.5, and 1.1.6) and Fixed Assets Accounts (Tables 1.1, 1.2, 5.1, and 5.2) of the Bureau of Economic Analysis.

[28] The data on home real estate and home mortgages are obtained from Table B.100 in Flow of Funds Accounts. A model-implied share of home mortgages in total loans ends up with 47% at the steady state.

are priced by the most leveraged households because debt enables them to increase their bidding power. The resulting calibration thus helps to capture part of the claim of Geanakoplos (2010).

Lastly, the discount factor of impatient households $\beta^I$, the parameter concerning dividends from banks to patient households $\delta_b$, and the transfer received by entrepreneurs $W^e$ are endogenously determined to ensure that the related steady-state equations hold. The resulting $\beta^I$ is 0.9598.

The bottom panel of Table 3 reports some selected model-implied steady-state ratios and their empirical counterparts, if they exist. The corresponding data are sample averages computed over the period of 1991-2006. Overall, the model matches quite well many of empirical moments that are not targeted. They include financial variables such as charge-off rates for home mortgages and business loans, the interest rate spread of mortgage lending rate over risk-free rate, and the interest rate spread of business loan rate over risk-free rate.[29] The resulting aggregate loan to value ratio of households (0.31) is a bit short of the empirical moment (0.37). It could reflect that part of home mortgages can be also issued to patient households, in reality.

[29]Regarding charge-off rates, the corresponding empirical counterparts are charge-off rates on single family residential mortgages of all commercial banks and those on business loans of all commercial banks, respectively. Charge-off rates are defined as loans removed from banks' balance sheets and charged against loss reserves, net of recoveries as a percentage of average loans in annual terms. I define the mortgage rate spread as the yield on the 30-year fixed rate mortgages minus the average yield on the 5-year and 10-year Treasury bonds, following Walentin (2014). He proposes this definition based on the observation that the duration of a 30-year fixed rate mortgages is, on average, 7-8 years. The interest rate spread of business loans is defined as the difference in the yield on Moody's Baa-rated seasoned corporate bonds and the 10-year Treasury bonds. All the date are obtained from FRED. More detailed data sources are presented in footnote 40.

Table 3: ENDOGENOUSLY DETERMINED PARAMETERS AND SELECTED STEADY-STATE RATIOS

| *A. Endogenously determined parameters* | | |
|---|---|---|
| Description | Parameter | Value |
| Discount factor, impatient households | $\beta^I$ | 0.9598 |
| Housing weight in utility, patient households | $\chi^P$ | 0.032 |
| Housing weight in utility, impatient households | $\chi^I$ | 0.190 |
| Capital share in production | $\alpha$ | 0.307 |
| Commercial real estate share in production | $1-\nu_k$ | 0.050 |
| Depreciation rate | $\delta_k$ | 0.0226 |
| Steady-state default threshold, home mortgages | $\bar{\omega}$ | 0.850 |
| Standard deviation, idiosyncratic shock on households' housing | $\sigma_\omega$ | 0.060 |
| Steady-state default threshold, loans to entrepreneurs | $\bar{\omega}_e$ | 0.316 |
| Standard deviation, idiosyncratic shock on entrepreneurs' capital | $\sigma_{\omega_e}$ | 0.419 |
| Bank net worth transferred to patient households | $\delta_b$ | 0.005 |
| Transfer received by Entrepreneurs. | $W^e$ | 0.0093 |
| *B. Steady-state ratios* | | |
| Description | Model | Data |
| *(Targets)* | | |
| Home real estate to output ratio, $\frac{Q_h(H^P+H^I)}{Y(=C+I)}$ | $1.7\times 4$ | $1.7\times 4$ |
| Share of home mortgages in total loans, $\frac{Z^I}{Z^I+Z^E}$ | 0.47 | 0.45 |
| Nonresidential fixed assets to output ratio, $\frac{K}{Y}$ | 6.5 | 6.5 |
| Nonresidential investment to output ratio, $\frac{I}{Y}$ | 0.147 | 0.146 |
| Share of business residential fixed assets, $H^E$ | 0.11 | 0.11 |
| Loan to value ratio of credit-constrained households, $\frac{R^L Z^I}{Q_h H^I}$ | 0.85 | 0.85 |
| Entrepreneurs' debt to net worth ratio, $\frac{Z^E}{N^E}$ | 0.46 | 0.42 |
| Annual foreclosure rate, $F(\bar{\omega})$ | 1.5% | 1.5 % |
| Annual business default rate, $F(\bar{\omega}_e)$ | 2.24% | 2.25 % |
| *((Model-implied steady-state ratios)* | | |
| Capital to output ratio, $\frac{K+Q_h H^E}{Y}$ | 7.3 | 7.1 |
| Charge-Off rates, households, $400(F(\bar{\omega})Z^I-(1-\mu^B)G(\bar{\omega})Q_h H^I)$ | 0.27 | 0.15 |
| Charge-Off rates, entrepreneurs, $400(F(\bar{\omega}_e)Z^E-(1-\mu^E)G(\bar{\omega}_e)(R^k K+R^h Q_h H^I))$ | 0.68 | 0.74 |
| Households' aggregate loan to value ratio, $\frac{R^L Z^I}{Q_h(H^P+H^I)}$ | 0.31 | 0.37 |
| Interest rate spread of home mortgages in annual terms, $R^L-R$ | 1.8%p | 1.8%p |
| Interest rate spread of business loans in annual terms, $R^E-R$ | 2.2%p | 2.1%p |
| Nominal risk-free rate in annual terms, $R^n=R\cdot\Pi$ | 4.0% | 4.1% |
| Banks' interest rate spread in annual terms, $R^r-R$ | 1.5%p | - |

# 4 Results

This section presents the findings of a series of simulation exercises. I begin with an impulse response analysis with two financial shocks: a housing preference shock and a shock to bank losses. Both shocks were identified as critical during the Great Recession. The simulations reveal the key mechanism of the baseline model. The subsequent analysis deals with policy experiments starting from the steady state, that is, policy experiments in non-crisis times. The government levies lump-sum taxes on patient households and uses them either to increase banks' capital or to reduce impatient households' existing debt. Next, I conduct a crisis experiment to mimic key features of the U.S. housing debt crisis. To be more specific, I feed a sequence of negative housing preference shocks into the model so that the model can replicate the observed decline in housing prices, and take a look at the isolated effects of such a collapse in housing prices on other macro and financial variables. Then, such an environment is used as a laboratory for comparing the consequences of two policies, the capital injection to banks and the debt relief to households. The magnitude and timing of each policy are chosen to be comparable to those of financial sector support programs that the U.S. government conducted during the recent crisis (see Table 1). Last but not least, I analyze how the consideration of the zero lower bound (ZLB) can affect the relative effectiveness of both policies.

In simulating the model, I first compute the non-stochastic steady state where all of the endogenous variables remain constant and the empirical targets are matched. Then I compute the approximated time path of endogenous variables in response to an exogenous shock (or shocks) by log-linearizing the model's equilibrium conditions around the non-stochastic steady state.

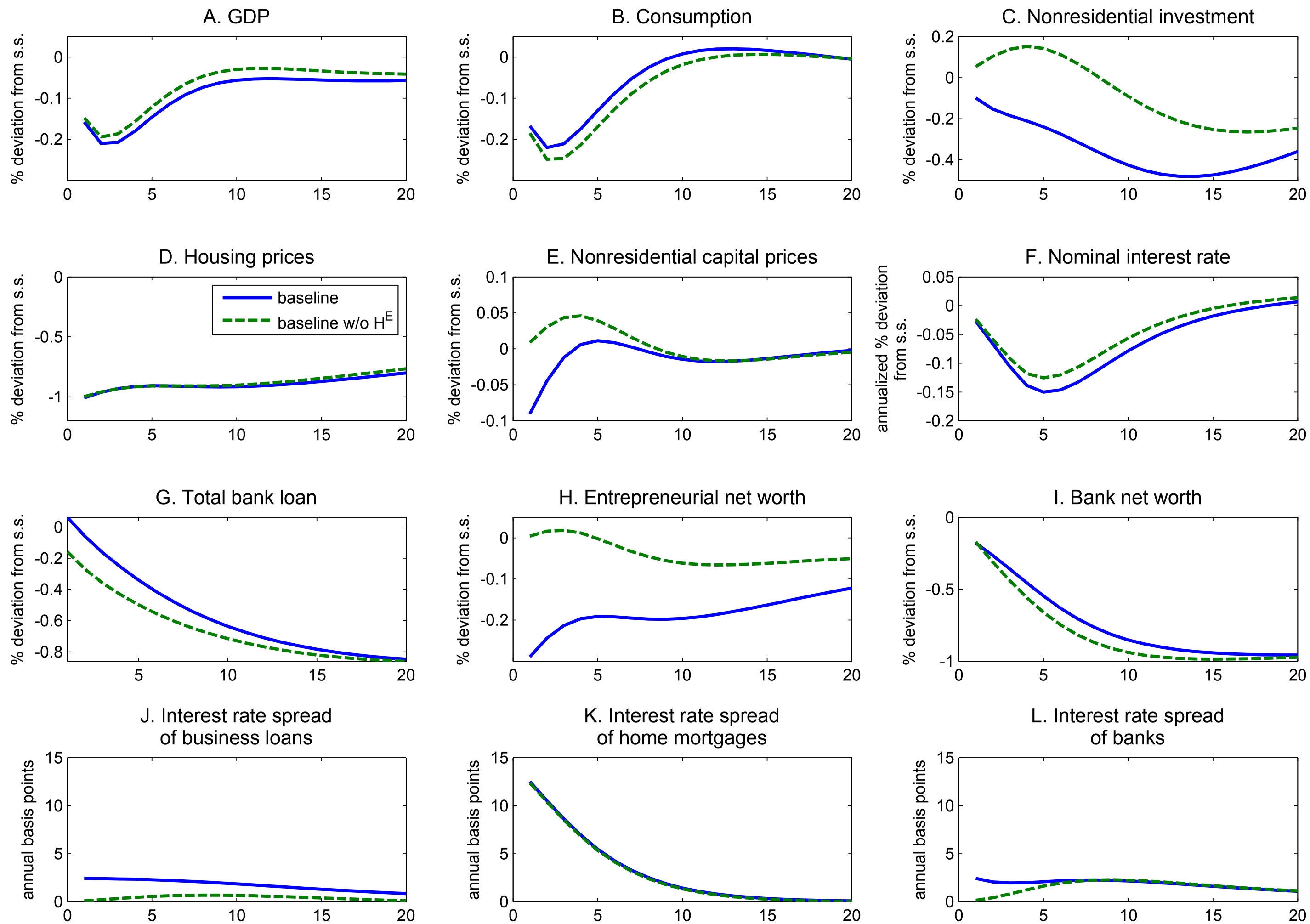
## 4.1 Baseline Simulations

### 4.1.1 A housing preference shock

Figure 2 presents dynamic responses to a negative housing preference shock that decreases the real housing price on impact by 1%. In order to highlight the role of commercial real estate in the model dynamics, Figure 2 also contains simulation outcomes from a model without commercial real estate $H_t^E$. Entrepreneurs in this alternative model economy manage only nonresidential capital as in the standard financial accelerator model in Bernanke et al. (1999).[30] I first look at simulation results from the baseline model, the solid blue lines, and then compare them with those from the model without $H_t^E$, the dashed green lines.

[30]Technically, I set $\nu_k$ at 0.000001, while ignoring a share of business residential capital as a target.

Figure 2: IMPULSE RESPONSES TO A NEGATIVE HOUSING PREFERENCE SHOCK IN THE MODELS WITH AND WITHOUT COMMERCIAL REAL ESTATE




*Notes*: Horizontal axis represents quarters after the shock. The simulations show dynamic responses to a housing preference shock that decreases housing prices on impact by 1% in the both models.

As seen in Panel D, housing prices fall immediately by 1% and then very slowly return to the steady state. Given the outstanding debt, the decline in housing prices increases borrowers' leverage. It implies that more members of impatient households and entrepreneurs default on their loans. The increased number of defaults in turn causes losses for banks. In order to satisfy their participation constraint, banks raise the contractual lending rates relative to the wholesale lending rate. The interest rate spread of home mortgages (in Panel K) goes up by 12 annual basis points(bps), while that of business loans (in Panel L) increases by 2 annual basis points. Since commercial real estate is part of entrepreneurial assets, the effects of housing prices on the entrepreneurial net worth are smaller than those on the net worth of impatient households. Facing higher borrowing costs, borrowers demand less credit. Meanwhile, a fall in housing prices also tightens credit supply, as the bank net worth (in Panel I) declines and the bank leverage rises. As a result, the interest rate spread of banks (in Panel L) increases by 2 annual basis points.

Given higher borrowing costs, the impatient households significantly cut back on consumption, making aggregate consumption decrease by roughly 0.2% (in Panel B).[31] The entrepreneurs also reduce demand for nonresidential capital, which leads to a decline in investment as well as in its price (see Panels C and E). Interestingly, the decrease in housing prices has a persistent effect on nonresidential investment. The trough is reached in the third year after the shock. As shown in Panel A, GDP, defined as the sum of aggregate consumption and nonresidential investment, decreases by about 0.2%. In response to the decrease in output and inflation (not shown in Figure 2), the central bank decreases its nominal interest rate (in Panel F).

Now I compare simulation outcomes from the baseline model with those from a model without residential capital $H_t^E$. Noticeable differences are found in the dynamic responses of nonresidential investment, its prices and the entrepreneurial net worth (in Panels C, E, and H). Since entrepreneurs do not deal with residential capital in the alternative model, housing prices cannot directly affect the entrepreneurs' net worth. Rather, the reduction in the equilibrium real interest rate, which is induced to compensate the reduced consumption of impatient households, boosts investment demand and thus raises the price of nonresidential capital. Nonresidential investment and its prices increase for nearly two years before they fall below the steady-state mainly due to the halting credit supply.

The simulation using the baseline model shows that a decline in housing prices leads to a decrease in nonresidential investment, which is in line with the empirical evidence presented in Liu et al. (2013). In contrast, the simulation with the alternative model, where a standard modeling setup for housing services like in Iacoviello (2005) or Iacoviello and Neri (2010) is simply combined with a financial accelerator mechanism for new business investment like in Bernanke et al. (1999), predicts the opposite with respect to housing prices and nonresidential investment.[32] Due to a similar logic a decline in housing prices leads to a business investment boom in a model

[31] The consumption of patient households increases due to a decline in the equilibrium real interest rate.

[32] Kollmann et al. (2013) and Clerc et al. (2015) simply combine the two macro-financial modelling devices. Regardless of whether or not they recognize their model's implications on housing prices and business investment, they rightly exclude a housing preference shock from a list of exogenous shocks considered.

where credit-constrained households borrow funds against housing collateral and unconstrained households can only access investment technology.[33]

### 4.1.2 A shock to bank losses

I investigate the effects of capital shortfalls in banks on the economy. Such a disturbance directly exacerbates credit supply conditions. To evaluate the implications of bank capital losses in the baseline model in contrast to those in other macro-financial models, I conduct the same simulation exercise with a shock to bank losses as in Guerrieri et al. (2015).

They model a shock to bank losses as a lump-sum transfer from the banking sector to households, who are the ultimate suppliers of funds in the economy, and assume that the shock follows an AR(1) process with an autocorrelation coefficient of 0.9. An exogenous disturbance is then fed into the model, so that the banking sector incurs cumulated losses worth 7.5% of annual steady-state GDP for 9 quarters.[34] Guerrieri et al. (2015) carry out the described simulation with five macroeconomic models, all of which consider an explicit role of banks and have been developed by staff economists at the Federal Reserve Board. They include the model of Iacoviello (2015), the model of Covas and Driscoll (2014), the model of Kiley and Sim (2015), Queralto's model, and the model developed by Guerrieri and Jahan-Parvar. Each model takes a different approach to formulate macro-financial linkages, but all of them are calibrated or estimated on U.S. data. The authors therefore argue that the model comparison can offer a "model-based confidence interval" with respect to the shock which originated from the banking sector.

Figure 3 presents the implications for a transfer shock from the banking sector to patient households in my model.[35] As the transfer shock causes the bank's net worth to decrease, the bank's interest rate spread rises for any given level of credit to borrowers.[36] As can be seen in Panel E, the bank net worth initially falls by about 10%, and then declines further by over 30% in 6 quarters due to an endogenous decrease in the bank's retained earnings as well as the persistence of the shock process. Thereafter it gradually returns to the steady state. The bank's interest rate spread shows symmetric dynamics. It increases to more than 2.0%p in annual terms and then steadily returns to the long-run level. With the soaring funding costs, entrepreneurs reduce the demand for capital and, as a result, nonresidential investment decreases rapidly as shown in Panel C. For

[33]In the models of Justiniano et al. (2015) and Iacoviello and Neri (2010), business investment increases in response to a negative housing preference shock because of these modeling assumptions.

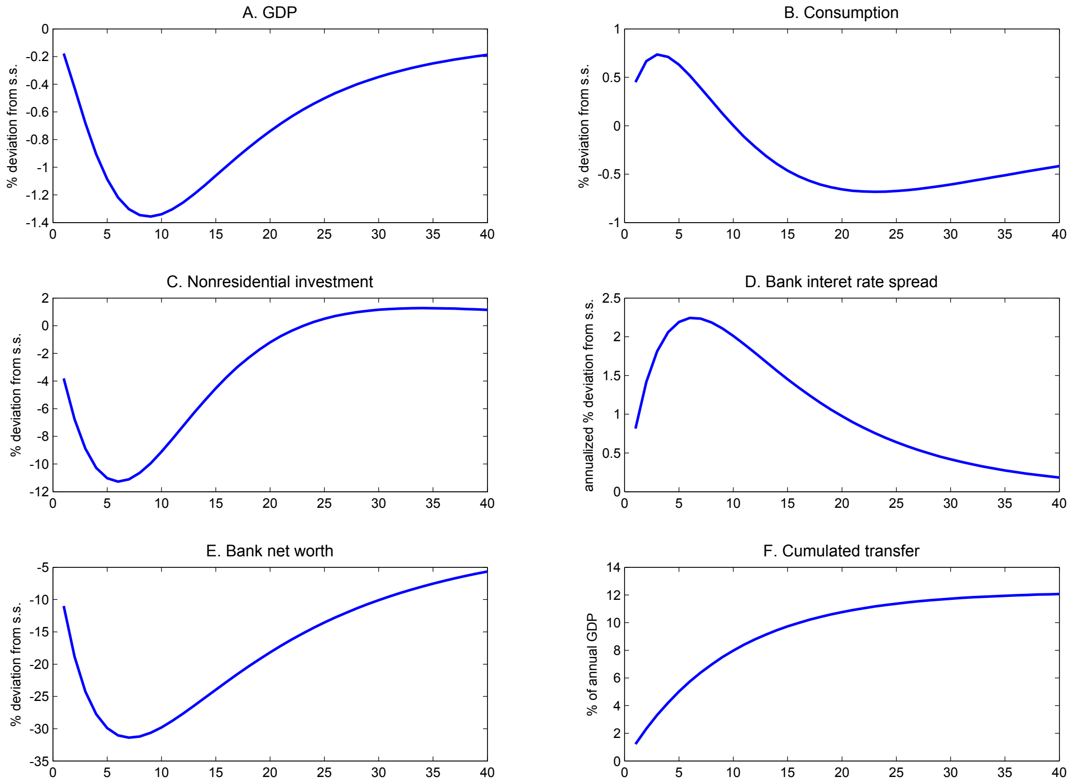
[34]This numerical reference is chosen based on the stress tests for the U.S. banking sector under a severely adverse scenario comparable to the Great Recession, which were conducted for the Comprehensive Capital Analysis and Review (CCAR) of 2013. The amount of the cumulated losses, 7.5% of annual GDP, is used to pin down the magnitude of an initial disturbance.

[35]To make the direct comparison easier, Figure 3 includes the same variables as in Figure 17, page 48 of Guerrieri et al. (2015).

[36] The log-linearized first order condition of the bank around the steady state shows this relationship:

$$\hat{R}_t^r - \hat{R}_t = \frac{\phi_n \nu_b^3}{R}\hat{W}_t - \frac{\phi_n \nu_b^3}{R}\hat{N}_t. \tag{32}$$

Figure 3: IMPULSE RESPONSES TO A TRANSFER SHOCK FROM THE BANKING SECTOR TO PATIENT HOUSEHOLDS




*Notes*: Horizontal axis represents quarters after the shock. The exogenous shock to transfer resources from the banking sector to patient households is fed into the model, so that the banking sector incurs cumulated losses worth 7.5% of annual steady-steady GDP for 9 quarters. This figure is directly comparable to Figure 17 of Guerrieri and Iacoviello (2015a).

the same reason impatient households reduce consumption. However, patient households, who receive a persistent wealth transfer from the banking sector and face a lower interest rate on deposits, increase consumption enough to offset the decrease in the consumption of impatient households. The aggregate consumption therefore goes up for the first two years (see Panel B). To sum up, the aggregate demand, or GDP, decreases by nearly 1.4% in the two years after the initial shock occurs and then gradually returns to the steady-state level.

Compared to the output dynamics of the five macro-financial models in Guerrieri et al. (2015), GDP in my model lies in mid-range. This comparison suggests that my model delivers quantitatively acceptable implications for the role of banking capital.

### 4.1.3 Policy experiments in normal times

So far, I have analyzed dynamic responses to two financial shocks and have found that the leverage of both borrowers and banks plays an important role in the model dynamics. When the leverage of impatient households or entrepreneurs increases, the corresponding lending rate spread also goes up reflecting a rise in default. Faced with higher funding costs, credit-constrained agents cut back on spending. Meanwhile, when the bank's leverage increases, the bank reduces loan supply, thereby increasing the wholesale loan rate relative to the rate at which the bank borrows. This adverse credit supply leads credit-dependent agents to refrain from spending.

Against the backdrop of these adverse disturbances, I carry out policy simulations to reduce the leverage of credit-constrained agents. More specifically, I compare the effects of a policy to inject capital into banks with those of a policy to reduce the outstanding debt of impatient households. For simplicity, I model each policy as a one-time transfer worth 1% of the steady-state annual GDP from the patient households either to banks or to the impatient households. In other words, the government earns funds by levying lump-sum taxes on patient households and uses them to support the financially-constrained agents. A detailed explanation and mathematical derivations are shown in Appendix D. The transfer to banks simply increases the banks' net worth. The policy to reduce the debt of impatient households needs more explanation. This policy involves two effects on the recipients. It increases their net worth like a simple transfer. On top of that, it scales down their leverage and thus reduces the likelihood of default. Thanks to the reduced foreclosure costs, banks make profits and will charge a lower mortgage rate relative to the wholesale lending rate, which relieves households of an interest repayment burden. Therefore, they have more incentive to increase current consumption. For comparison, I additionally consider a simple lump-sum transfer to impatient households.

Figure 4 shows the dynamic responses to three one-time transfer policies worth 1% of steady-state annual GDP starting from the steady state: a transfer to banks (solid blue line), a debt relief to impatient households (dotted red line), and a transfer to impatient households (dashed green line). The transfer to banks increases credit availability substantially, so that the interest rate spread of banks decreases by around 70 annual basis points and total bank loans increase on impact by almost 2% (see Panels L and G). Consequently, consumption and nonresidential investment increase. The impact on investment (in Panel C) is large. The increased investment demand causes capital prices to rise, and thus entrepreneurial net worth also goes up. This further boosts nonresidential investment in the subsequent periods. In contrast, the impact on aggregate consumption is fairly mild. The consumption of impatient households gradually increases partly due to mortgage adjustment costs and is mostly offset in the aggregate by a decrease in the consumption of patient households. Housing prices decline as patient households reduce their demand for housing services due to negative wealth effects. The effects on inflation are negligible and thus the central bank raises the nominal rate only a little.

Overall, real GDP increases by 0.17% before slowly returning to its long-term level. The output effects are quite persistent: GDP increases by 0.14% for each of the first two years and by 0.08% for the third year. The cumulative output increase after ten years amounts to roughly 0.6% relative to the baseline annual GDP.

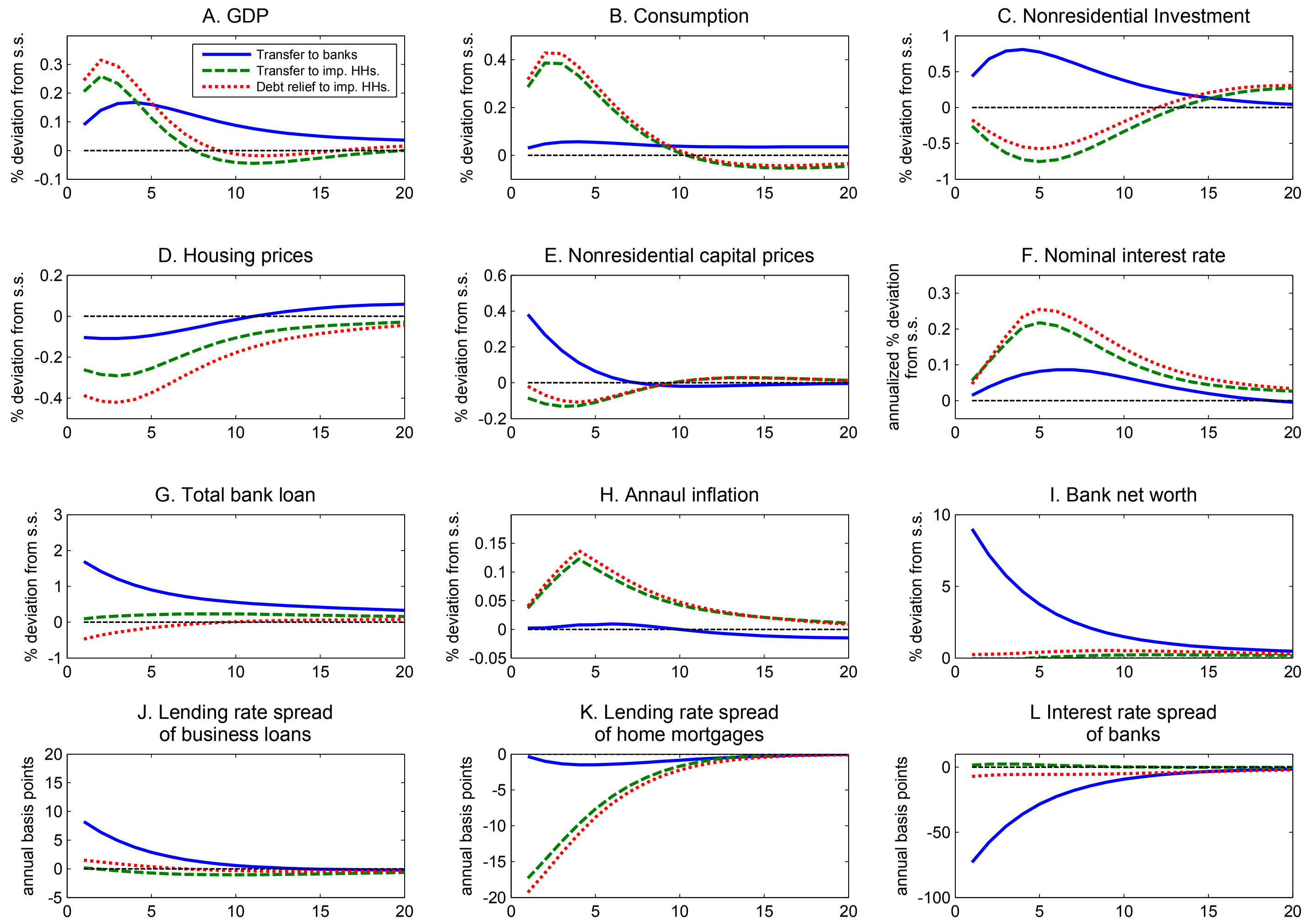
I now turn to evaluating a policy to reduce the debt of impatient households with funds raised from patient households. The marginal propensity to consume of the impatient households is higher than that of the patient households due to a lower discounting factor, so that a simple transfer, by itself, can lead to an increase in aggregate consumption. The dashed green line shows the simulation outcomes with the lump-sum transfer to impatient households. Yet, the debt relief has an additional channel to increase consumption. The reduction in existing debt results in a lower funding cost for new home mortgages through the decreased mortgage default risk. The lending rate spread of home mortgages decreases on impact by 20 annual basis points, which is 2 basis points larger relative to those under the policy with a simple transfer. Consequently, aggregate consumption increases initially by 0.3% and then goes up by 0.43% in the next period, which is 0.05% higher than with the policy using a simple transfer.

However, this sharp boost in consumption leads to a rise in inflation. In response to such an inflationary development, the central bank raises the nominal interest rate, which translates into a rise in the real interest rate due to price rigidities. A higher real rate causes patient households to cut back on consumption further, so that 10 quarters after the policy is undertaken, aggregate consumption falls below its steady state (see Panel B). It also leads entrepreneurs to refrain from investment (in Panel C) and thus nonresidential capital prices decline (in Panel E). It is particularly interesting to see that housing prices (in Panel D) decline more under debt relief than under capital injections to banks, even though debt relief increases the impatient households' appetite for housing services. This is because under this policy, patient households reduce housing demand to a greater extent due to higher real interest rates in addition to negative wealth effects.

Overall, real GDP increases sharply by nearly 0.3% in the first year and then slows down to 0.1% in the second year. Then it falls below the steady state and turns negative in the third year and thereafter returns to the non-policy path. In the case of the policy with a simple transfer, stimulative output effects are more short-lived. The cumulative increase in output after ten years reaches 0.45% over baseline annual GDP.

To sum up, the output effects of a debt relief to credit-constrained households are roughly twice as large as those after injecting resources to banks in the first year. However, the latter dominates the former from the second year onward, thereby making a transfer to banks more effective over the long run. It is because such a transfer causes nonresidential investment to increase, expanding production capacity and thereby resulting in lower inflation. On the contrary, household debt relief gives rise to higher inflation and therefore leads to the central bank's reaction to raise the interest rate, which significantly dampens investment as well as the consumption of credit-unconstrained households.

Figure 4: EFFECTS OF ONE-TIME TRANSFER POLICIES WORTH OF 1% STEADY-STATE ANNUAL GDP




*Notes*: Horizontal axis represents quarters after each policy shock.

## 4.2 An application to the U.S. housing debt crisis

### 4.2.1 The effects of the collapse in housing prices

In this subsection I analyze the effects of the sharp decline in house prices observed in the U.S. subprime mortgage crisis. House prices started to decline from the first half of 2006, long before the recession began, and plummeted until the second quarter of 2009. Then they remained more or less flat until 2011 before they started to recover. Based on the S&P Case-Shiller U.S. National Home Price Index adjusted by the implicit GDP deflator, house prices dropped by around 30% from 2006Q1 to 2009Q2. If the stochastic trend is removed, house prices declined by nearly 37% relative to the long-run trend over the same periods.[37]

To engineer such a fall in housing prices, I feed a series of negative housing preference shocks into the model, similarly to the approach taken by Justiniano et al. (2015). This strategy to generate the decline in housing prices is based on several studies which, using an estimated structural model with housing, document that shocks on housing preferences are the primary determinant of fluctuations in housing prices. Liu et al. (2013) show that housing preference shocks explain around 90% of fluctuations in housing prices at all the different forecasting horizons from one quarter to six years.[38] Also, Guerrieri and Iacoviello (2015a) find that the decline in housing prices observed during the Great Recession is almost exclusively explained by the realizations of housing preference shocks.[39] Therefore, this exercise allows us to quantify the isolated effects of the observed decline in housing prices on macroeconomic aggregates as well as the relevant financial variables.

Figure 5 presents the model-simulated outcomes together with their analogous data observed from the first quarter of 2006 to the fourth quarter of 2014. The actual data includes GDP, consumption, nonresidential investment, the sum of home mortgages and business credit, housing prices, the federal funds rate, charge-off rates on single family residential mortgages, charge-off rates on business loans, and total equity to total assets for all commercial banks.[40] The aggre-

[37] I use a one-sided HP filter with a value of $\lambda = 100,000$ to decompose a time series into a trend component and a cycle component. Guerrieri and Iacoviello (2015a) argue that choosing high $\lambda$ produces plausible estimates for macroeconomic aggregates and financial variables during the Great Recession. In addition, the one-sided filter prevents the decomposition from being influenced by the correlation between the current observation and the subsequent observations.

[38] The finding is based on variance decompositions reported in Table 3 of Liu et al. (2013). Their model includes a representative patient household and a representative impatient entrepreneur. The entrepreneur's borrowing is tied to the value of collateral assets consisting of commercial real estate and nonresidential capital. The model employs eight exogenous shock processes: an intertemporal preference shock, a housing preference shock, a labor supply shock, permanent and transitory technology shocks, permanent and transitory investment-specific shocks, and a collateral shock.

[39] The results are obtained from historical decompositions presented in Figure 5 of Guerrieri and Iacoviello (2015a). The model includes patient households and impatient households. The debt of impatient households is occasionally tied to their housing value. The model uses six exogenous shock processes: an intertemporal preference shock, a housing preference shock, a price markup shock, a wage markup shock, an investment shock, and a monetary policy shock.

[40] I obtain data series on consumption and nonresidential investment from U.S. National Economic Accounts. Using the chain-aggregation methods, I compute the model-consistent output, defined as the sum of consumption and nonresidential investment. The rest of data can be downloaded from FRED (https://research.stlouisfed.org/fred2/): Home Mortgages (HMLBSHNO), Credit Market Instruments for Nonfinanical Corporate Business (TCMILBSNNCB), Credit

gate quantity variables are expressed in real, per capita terms.[41] These variables in addition to housing prices are detrended using a one-sided HP filter (with a value of $\lambda = 100,000$) in order to remove the low-frequency components.[42] Each data series in Figure 5 is normalized such that its estimate for the first quarter of 2006 is at zero. The rest of the variables are displayed without transformation. The model-simulated series starts from their steady-state values. GDP, consumption, nonresidential investment, housing prices, and total real loans are expressed in the percentage deviation from their own steady state. The remaining variables are measured in the same way as their data counterpart.[43]

Innovations in housing preferences are introduced into the model for 13 quarters in a row, such that the model can replicate the historical development of housing prices during the periods from 20061Q to 2009Q2. As can be seen in Panel E, the model-implied housing prices do indeed plummet as much as the actual data over the same period. Once no more innovations are fed into the model, the model-implied housing prices gradually recover to the long-run level, which is broadly in line with the actual data. With such a collapse in housing prices, defaults on home mortgages soar and, consequently, so do bank losses. The annualized charge-off rate on home mortgages rises from 0.1% to more than 2.5% for the initial 13 quarters (in Panel G). This surge accounts for over 90% of the peak in the actual charge-off rates. Another observation is that the model-implied charge-off rates are shifted to the left by around four to six quarters relative to the actual data. This is mainly because in the model, the foreclosure process is completed within a period, but in reality it takes one year or more for financial institutions to foreclose home mortgages. Taking into account an actual time lag for the foreclosure completion, the model-implied charge-off rates on home mortgages explain well the actual outcomes. The differences between the model-implied and empirical consumption mirror those between the model-implied and empirical charge-off rates, because a rise in defaults reduces consumption through the resulting higher lending costs. A comparison of consumption from the model and data suggests that the observed housing prices explain roughly 60% of the actual decline in consumption.

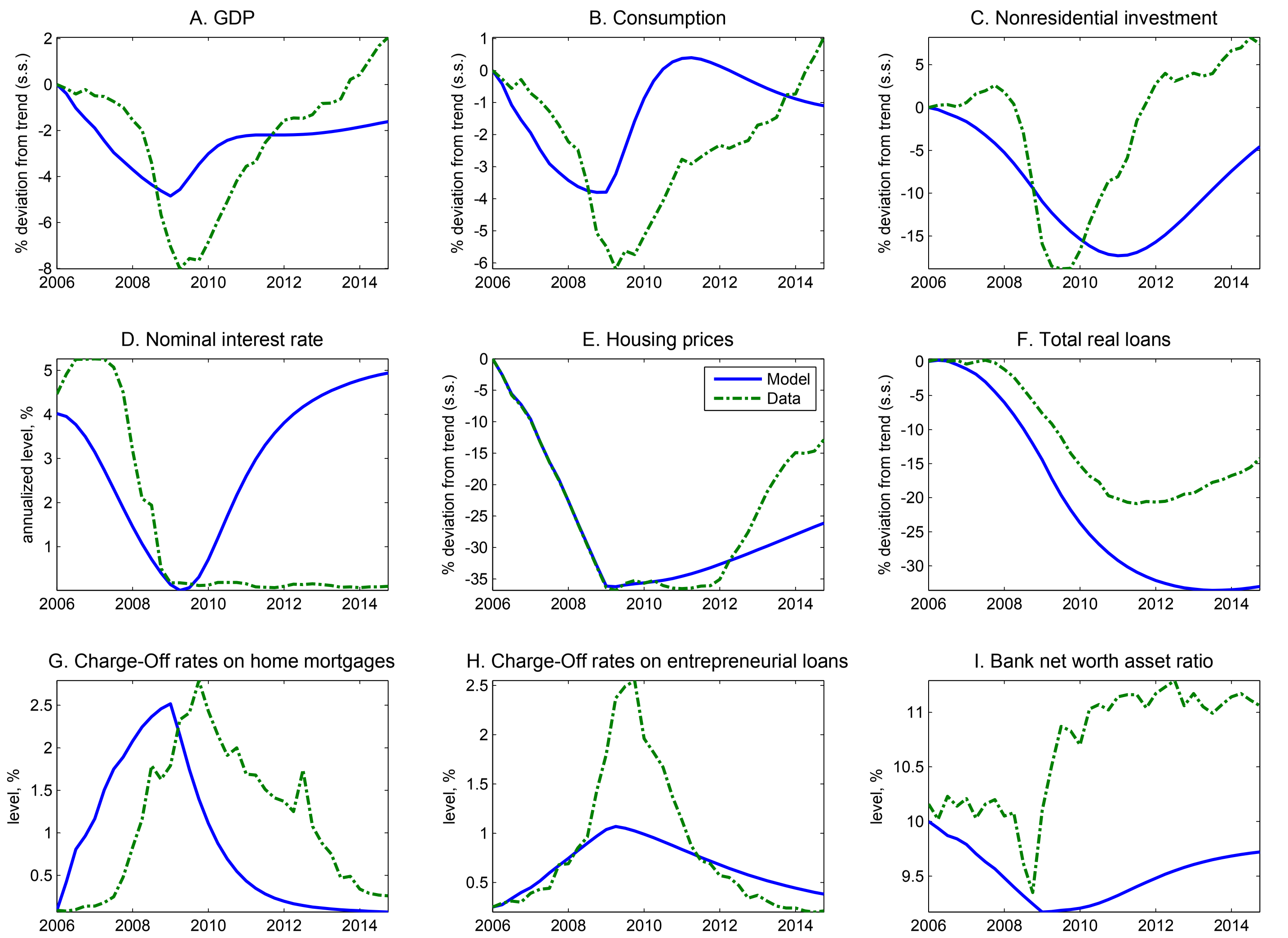
Market Instruments for Nonfinanical Noncorporate Business (TCMILBSNNB), S&P Case-Shiller U.S. National Home Price Index (CSUSHPISA), Effective Federal Funds Rate (DFF), Charge-Off Rate on Single Family Residential Mortgages (CORSFRMACBS), Charge-Off Rate on Business Loans (CORBLACBS), and Total Equity to Total Assets for Banks (EQTA). In parentheses is an ID for each data series in FRED. Total loans consist of home mortgages and credit market instruments for nonfinanical corporate business & nonfinanical noncorporate business.

[41]The credit variable is deflated by the GDP deflator. The variables are converted into per capita terms by dividing each by the working-age population, which is obtained from the Organization for Economic Cooperation and Development (OECD).

[42]I use quarterly data covering the period from 1985Q1 to 2014Q4.

[43]For a better comparison, the model simulations for the two charge-off rates start from the level of 2006Q1, not from the steady state.

Figure 5: AN APPLICATION TO THE U.S. HOUSING DEBT CRISIS (20061Q -20144Q)




*Notes*: For the data series, horizontal axis ranges from 2006Q1 to 2014Q4. GDP, consumption, nonresidential investment, total real loans and housing prices are shifted in vertical axis such that the estimates for 2006Q1 are at zero. The rest of data series are shown without transformation. The model-simulated series start from the steady state. Please refer to the text for detailed explanation.

The collapse in housing prices also increases defaults on entrepreneurial loans, thereby raising those loans' charge-off rates. Panel H suggests that other shocks must have increased charge-off rates on business loans during the Great Recession. One of them could be a shock to the second moment of an idiosyncratic shock process influencing the return to entrepreneurial capital as suggested in Christiano et al. (2014). Meanwhile, the observed decline in housing prices results in a delayed but substantial decrease in nonresidential investment due to the reduction of both credit demand and credit supply. Overall, the model predicts that lower housing prices observed during the periods from 2006Q1 to 2009Q2 are associated with a decrease in aggregate output below 5%. This accounts for 60% of the decline in output from the actual data.

In response to the development of GDP and inflation (not shown in Figure 5) induced by the collapse in housing prices, the central bank reduces the nominal interest rate by 4%p. Starting at a steady-state level of 4%, the nominal rate almost reaches zero (0.1%) 13 quarters after the initial innovation occurs. As losses in banks are accumulated, banks' net worth to asset ratios decline to roughly 9.2%. The drop in the model-implied bank net worth to asset ratio is comparable to that of its empirical counterpart. While the actual bank capital to asset ratio went up to over 11% from 2008Q4 after banks received the TARP funds from the government and stayed over 11%, the model predicts a fairly gradual returning to the long-run level. These differences occur mainly due to the model's inability to account for the post-crisis government responses and their impact on the value of banks' marketable assets. As for total real loans, we find that the deleveraging implied by the model is much faster than what had happened during the Great Recession.

#### 4.2.2 Policy experiments in the middle of the U.S. housing debt crisis

As presented in the introduction, the U.S. government used about $500 billion to recapitalize the U.S. financial system. This amount of funds corresponds to 4.2% of the sum of 2008 annual consumption and nonresidential investment. In this subsection, I analyze the effects of this recapitalization program and compare them with those of a debt relief program providing the same amount of funds. With the observation that almost all of the funds to support the financial sector were disbursed or committed in one or two quarters after the Troubled Assets Relief Program (TARP) was set up in September 2008, I model a policy intervention as a one-time unexpected shock, namely, a one-shot transfer worth 4.2% of annual GDP (consumption + nonresidential investment) from patient households to either banks or impatient households.[44] To focus on the effects caused by the policies, I abstract from a distortionary fiscal policy and assume that Ricardian equivalence holds.[45]

The results from the preceding policy exercises in normal times (in Section 4.1.3) imply that the bank recapitalization of 4.2% of annual GDP would increase GDP by nearly 1.2% for the first

[44] 99% of the TARP funds used to support the financial sector was disbursed or pledged from 2008Q4 to 2009Q1. The maximum commitment to rescuing Fannie Mae and Freddie Mac was initially announced in 2008Q4 and was revised once in 2009Q1.

[45] All lump-sum taxes are assumed to be paid by patient households. As they behave according to the permanent income hypothesis, the timing of taxes is irrelevant for their consumption-saving decisions.

two years, and by 1.3% for the next eight years. In case of the debt relief to impatient households, GDP would increase by 1.5% for the initial two years, followed by a mild increase (0.4%p) for the next eight years.[46]

These policy effects are simulated as a deviation from the model's steady state. Obviously, the U.S. economy was in the midst of severe financial turbulence when government funds were injected into financial institutions. Since a log-linearized version of the model is used for the simulation, a simple transfer policy would bring about identical results regardless of where the simulation starts. However, the effects of a debt relief can be different even in a linear model due to the introduction of default risk. The impatient households' decision whether to default or not depends on their outstanding debt. The amount of existing debt at any period during the simulation is not necessarily the same because it is endogenously determined as part of the equilibrium. On top of that, the federal fund rates have remained near zero since 2008Q4. Taking into account that the federal fund rates are constrained at the zero lower bound (ZLB) can create additional nonlinear effects on the economy. The subsequent experiments deal with these two considerations.

I again use the previous crisis experiment for evaluating the effects of the two policies. To measure policy effects I simulate the model with and without a policy shock and compute the differences in simulation outcomes. More precisely, I first compute the model implications only with a series of negative housing preference shocks, and then compute the model implications with an additional policy shock in 2009Q1 (the twelfth quarter).

Figure 6 depicts the effects of bank recapitalization (blue solid line) versus household debt relief (red dotted line) in the middle of the U.S. housing debt crisis. The left column shows the policy effects without the consideration of the ZLB. The vertical green line indicates the period when the policy action is undertaken. The effects of household debt relief on consumption are striking. The debt relief increases consumption more than 3%p relative to the consumption level that would prevail without the policy. In the experiment starting from the steady state, the same policy raises consumption up to around 1.8%p, which is far below 3%p. As shown in Panel G of Figure 5, charge-off rates on home mortgages are much more elevated in 2008Q4 and thus the lending rate spread of home mortgages is very high reflecting high default risk. Reducing the households' indebtedness in such an environment, by forgiving part of household debt, improves the lending conditions much more than in normal times. The debt relief causes a decrease in the annualized lending rate spread of home mortgages from 2.0%p to 0.6%p, while the same policy in normal times results in a decline of only 0.8%p. It also brings about further reductions in foreclosure costs. As a result, the aggressively lowered funding costs boost the consumption of credit-constrained households more. A surge in consumption, however, causes higher inflation. Due to the central bank's reaction, interest rates rise more, which in turn leads nonresidential investment to decline more than it does in normal times.

Overall, the household debt relief worth 4.2% of annual GDP increases GDP by 2.7%p for the

[46] Policy effects are computed by just multiplying the simulation outcomes given in Figure 4 with the relative magnitude of the policy (4.2).

first two years, and by 0.7%p for the next eight years. Due to a linear model, the effects of bank recapitalization, a simple transfer to banks, are the same as those in normal times: 1.2%p for the first two years and 1.3% for the next eight years. Therefore, the debt relief to credit-constrained households can have much stronger short-run stimulus on output when assuming that the economy is in the middle of a housing debt crisis.

### 4.2.3 Policy experiments with the consideration of the ZLB

I now examine how the consideration of the zero lower bound (ZLB) influences on the stimulative effects of both policies. In the experiment with regard to the U.S. housing debt crisis, the simulated nominal interest rate declines close to zero but never hits the ZLB. To increase the likelihood of the nominal rate's hitting the ZLB with few changes of the model dynamics in a linear model, I lower the steady-state annual inflation from 2% to 1%. I use a toolkit provided by Guerrieri and Iacoviello (2015b) to compute dynamic responses in the presence of the ZLB. The toolkit defines the regime based on whether the constraint binds or not, and uses a piecewise linear perturbation method that basically links the first-order approximation of the model around the same point under each regime. As the dynamics in each regime could depend on the expectation of how long the economy stays in that regime, the simulation outcomes can be highly nonlinear.

As in the previous policy experiment, I feed in each policy shock in the twelfth quarter. Without any policy, the nominal interest rate would stay at zero for five quarters more until the sixteenth quarter.[47] Figure 6 presents the simulation results for GDP, consumption, investment and the nominal interest rate without the ZLB on the left and with the ZLB on the right. First of all, the output effects of both policies are bigger when the nominal interest rate is constrained at zero than in the other case. The peak output effect of the transfer to banks under the ZLB is approximately 1.0%p, which is around 40% higher than that without the ZLB (0.7%p). The peak output effect of the debt relief to impatient households under the ZLB is roughly 4.0%p, which is nearly 70% higher than that of normal times (2.4%p). Second, each policy helps the economy to escape from a liquidity trap one quarter earlier. The nominal interest rate is constrained at zero for one year until the fifteenth quarter and thereafter it is set according to the interest rate rule. Third, the stimulative effects of the debt relief are magnified under the ZLB. The debt relief is inflationary, so that the real interest rate falls, when the nominal interest rate cannot go below zero. The decrease in real interest rates in turn boosts consumption of both households as well as nonresidential investment, and at least weakens crowding out effects.

Overall, a debt relief worth 4.2% of annual GDP increases annual GDP by 5.2%p for the first two years. These output effects are far beyond those of bank recapitalization (1.6%p). For the next eight years, the debt relief would increase GDP by 2.0%, whereas the bank capitalization boosts

[47]The nominal interest rate is endogenously kept at zero for two years from the ninth to sixteenth quarter. Since the ZLB prevents the nominal interest rate from falling down to the level at which efficient intertemporal allocations can be made, aggregate demand consisting of consumption and nonresidential investment decreases further than without the ZLB. The discussion and figures are presented in Yoo (2017).

GDP by 1.5%. These results suggest that the debt relief policy under the ZLB has far-reaching effects on the economy beyond the periods during which the nominal interest rate is constrained at zero. Table 4 summarizes the comparison of counterfactual output effects in three circumstances.

Table 4: OUTPUT EFFECTS OF EACH ONE-TIME POLICY WORTH 4.2% OF ANNUAL GDP

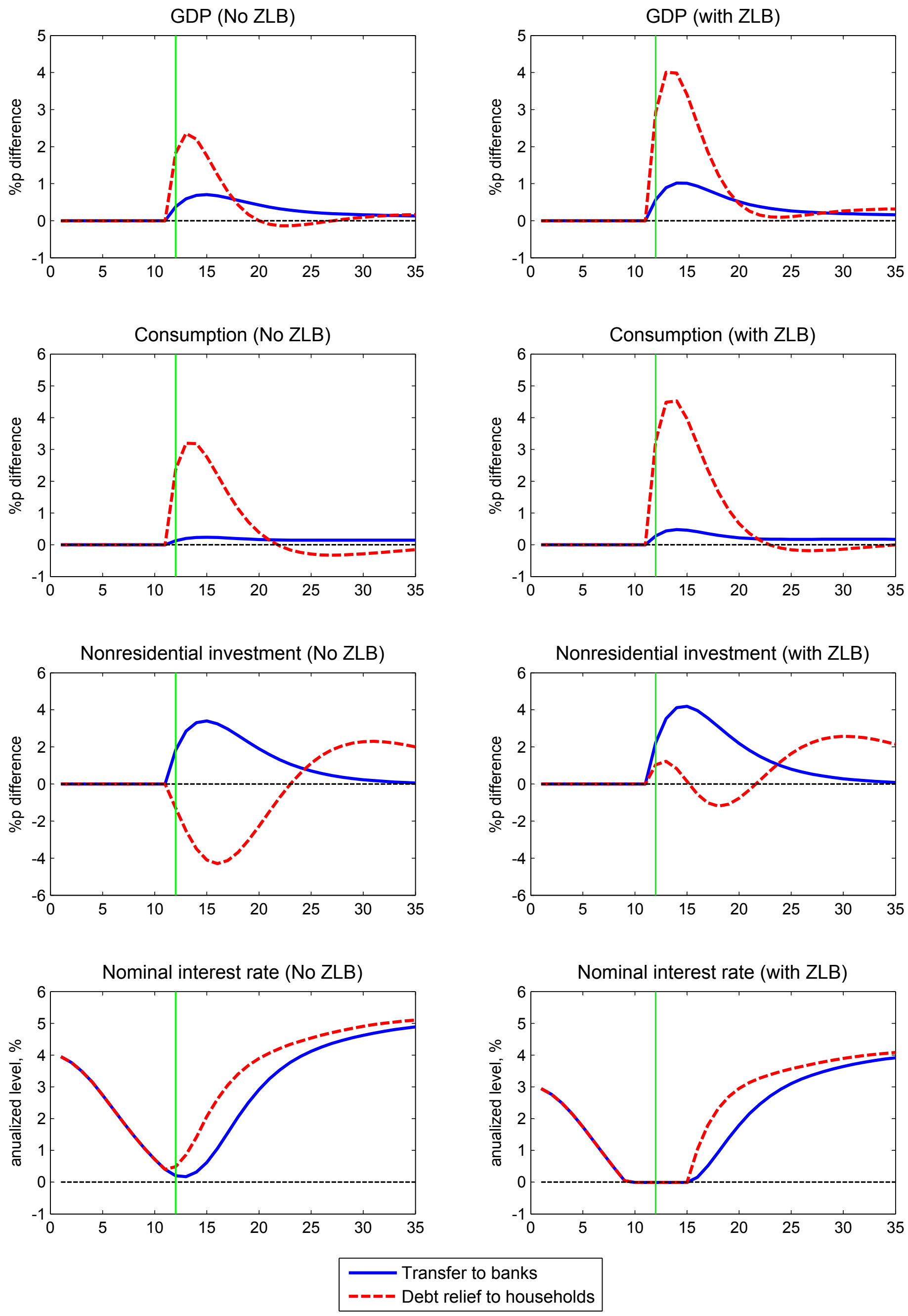
| | Short-run ($\sim$ 2 years) | | Long-run (3-10 years) | |
|---|---|---|---|---|
| | Bank recapitalization | Household Debt relief | Bank recapitalization | Household Debt relief |
| Normal times | 1.2 | 1.5 | 1.3 | 0.4 |
| Housing debt crisis | 1.2 | 2.7 | 1.3 | 0.7 |
| Housing debt crisis under the ZLB | 1.6 | 5.2 | 1.5 | 2.0 |

Note: output effects are measured relative to the baseline annual GDP.

I also conduct the same policy simulations with a different steady state inflation to show how the degree of monetary accommodation affects the stimulative effects of both policies. Figure 7 presents the cumulative output multipliers of each policy for ten years - the red line with squares for the transfer to banks and the green line with triangles for the debt relief to households - at each steady state inflation. Following Uhlig (2010), the cumulative multiplier is defined as a ratio whose numerator equals the present values of policy-induced output effects relative to the steady state for ten years, and whose denominator equals the size of the policy measure, which in this case is 4.2% of annual steady-state GDP. I assume zero discount rates, namely, making a simple sum of policy-induced output effects.[48] The number in the parenthesis on the x-axis indicates how many quarters the nominal interest rates are stuck at zero from the period of the policy intervention. As can be seen, the longer the policy rates are constrained at the ZLB, the higher the cumulative output multipliers are for both policies. The cumulative multiplier of capital injection policies is 0.6 when the ZLB is disregarded, but it doubles when the policy rate stays at zero for seven quarters. More interestingly, in the case of debt relief policies, the cumulative multiplier accelerates as the number of periods that the economy is constrained at the ZLB increases. The cumulative multiplier when the policy rate stays at zero for seven quarters is nearly 4.0, which is nearly five times as big as that without the ZLB (0.8).

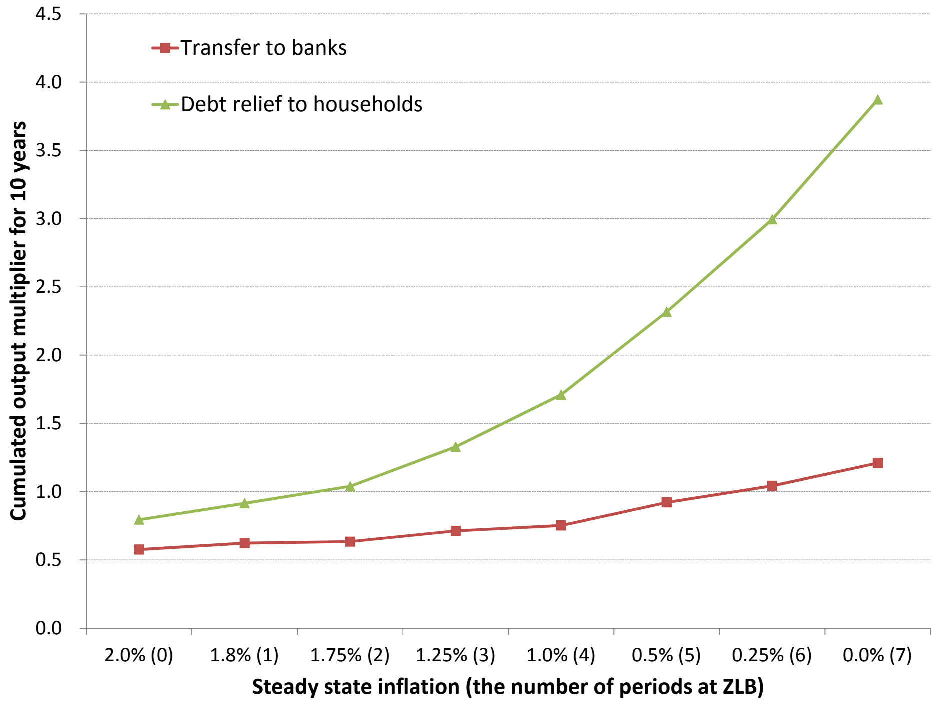
[48]This is equivalent to the assumption that the discount rates are the same as the trend growth rates.

Figure 6: EFFECTS OF BANK RECAPITALIZATION VERSUS HOUSEHOLD DEBT RELIEF WORTH 4.2% OF ANNUAL GDP IN THE MIDDLE OF THE U.S. HOUSING DEBT CRISIS




*Notes*: The simulations start from the steady state. The vertical axis represents the differences in model implications with a policy shock and those without a policy shock. The vertical green line in the twelfth quarter indicates the period when a policy action is undertaken. Please refer to the text for detailed explanation.

Figure 7: TEN-YEARS CUMULATIVE OUTPUT MULTIPLIERS AND THE DEGREE OF MONETARY ACCOMMODATION



# 5 Conclusion

In this paper, I build a DSGE model having major macro-financial linkages identified in the literature with a particular focus on the role of housing finance. I use the model to analyze the macroeconomic effects of a collapse in house prices observed in the recent U.S. housing debt crisis. Then such a crisis environment is used as a laboratory for assessing the relative effectiveness of a policy to inject capital into banks versus a policy to relieve households of mortgage debt. The magnitude and timing of each policy are chosen to be comparable to those of financial sector support programs that the U.S. government conducted during the crisis.

The policy simulation results suggest that policies to support the financial sector helped to ease the severity of the recession in the aftermath of the housing debt crisis. The more important implication is, however, that debt relief to households of the same amount of funds would have been much more effective to stimulate the economy, given that the household sector was highly leveraged and the nominal interest rates were constrained at the ZLB. Therefore, this paper supports the view of Mian and Sufi (2014) and others, who argue for a more aggressive reduction of household debt.[49]

Despite the detailed model structure, several issues need to be addressed in order to quantify the effects of household debt relief more precisely. First, it would take some time for government agencies to undertake the necessary steps to carry out a policy to reduce household debt, because such a policy involves a lot of parties unlike the financial sector support programs. In my policy experiments, however, the debt relief is modeled as a one-time policy intervention to facilitate a direct comparison with the capital injection to banks. As shown in Cogan et al. (2010), the significant implementation lag of fiscal policy can reduce its stimulative effects. Meanwhile, the anticipation of the scheduled debt relief policy can ameliorate the current credit condition by loosening the participation constraint of banks. In the end, it is a quantitative issue as to which one dominates the other. Second, while the model partly accounts for sluggish mortgage dynamics by introducing loan adjustment costs, the introduction of explicitly-defined long-term mortgage contracts would be more desirable. The recent papers by Gelain et al. (2015) and Elenev et al. (2015) deal with an interesting modelling setup for long-term mortgage contracts. Third, the default decision could be modeled in more detail. For example, by introducing a direct penalty or cost on defaulted agents an access to credit market would be restricted for a stochastic length of periods once borrowers declare default (see Chatterjee and Eyigungor (2015)).

Besides the evaluation of the post-crisis government policies, the model can be used or extended for analyzing other interesting macroeconomic policies. Areas for future research include the design of optimal monetary policy rules with financial variables (as in Curdia and Woodford (2010) , Gilchrist and Zakrajsek (2012), and Hirakata et al. (2013)), and the optimal bank capital regulation (as in Clerc et al. (2015) and Begenau (2016)).

[49]Note that throughout the policy simulations, I implicitly assume that the main driver of the recent housing debt crisis is a shock on house prices. If other shocks such as a shock originated from the financial sector is considered to be dominant, the relative effectiveness of the policies can be differed.

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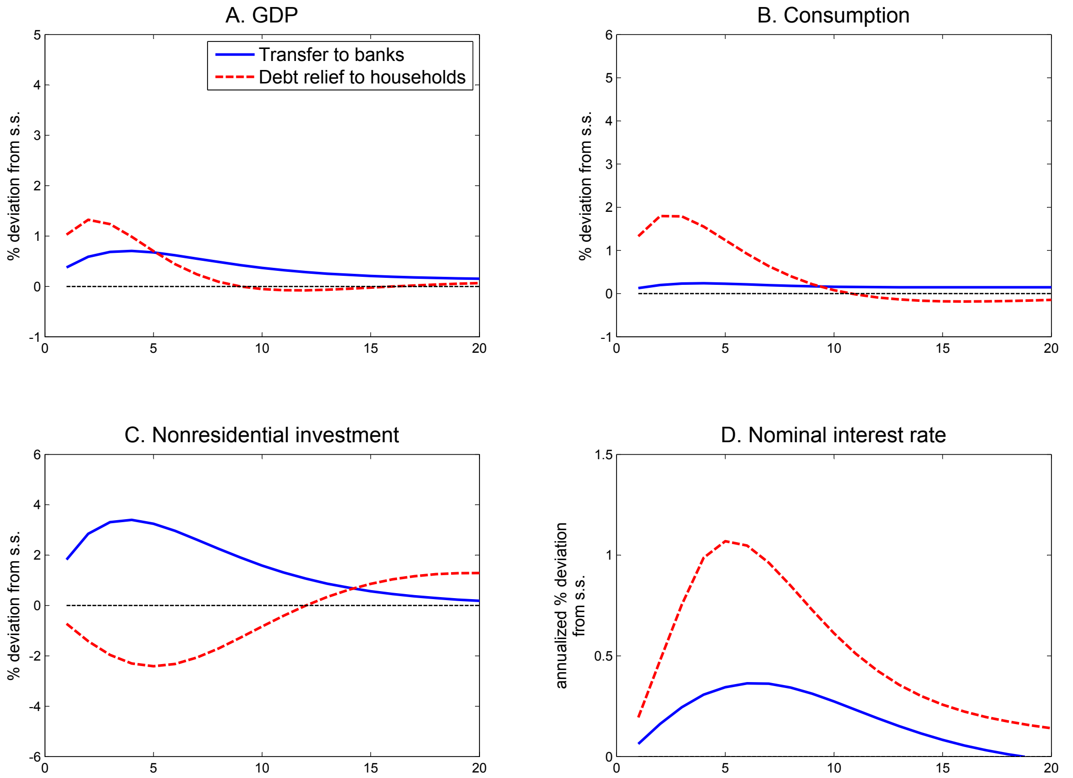
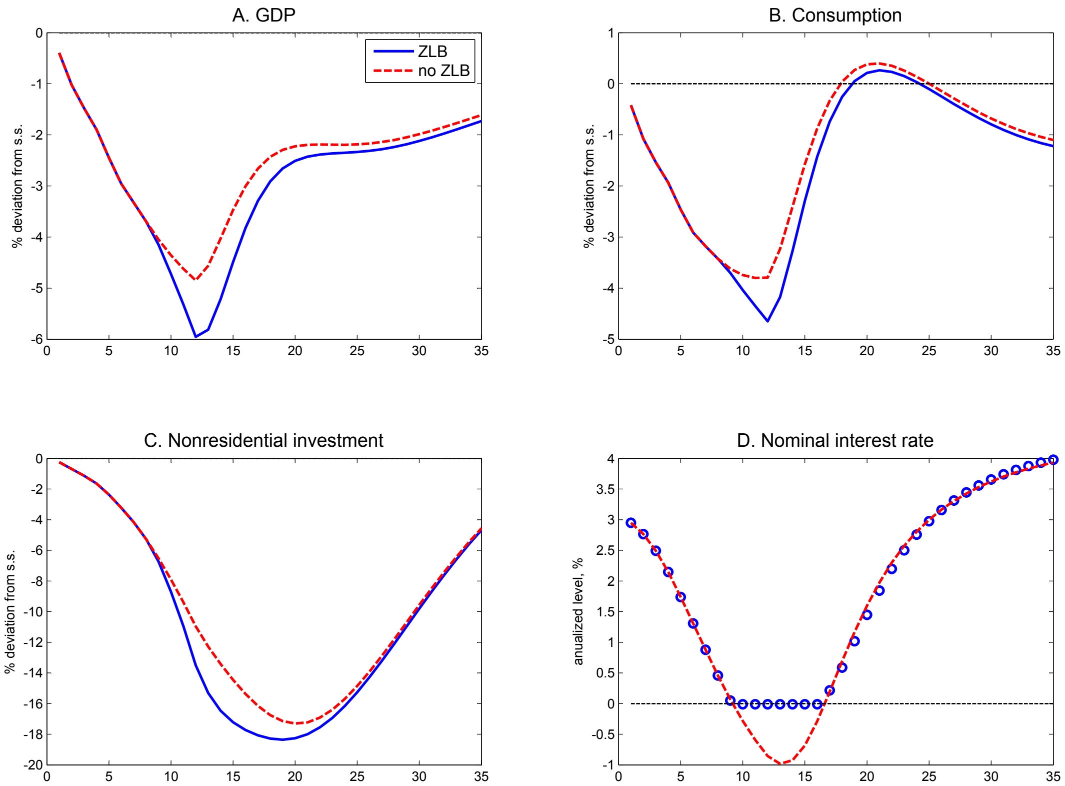
# A Additional figures

Figure A.1: EFFECTS OF BANK RECAPITALIZATION VERSUS HOUSEHOLD DEBT RELIEF WORTH OF 4.2% ANNUAL GDP IN NORMAL TIMES




*Notes*: Horizontal axis represents quarters after each policy shock.

Figure A.2: EFFECTS OF THE ZERO LOWER BOUND (ZLB) IN THE U.S. HOUSING DEBT CRISIS




*Notes*: The steady-state annual inflation is lowered from 2% to 1%. The simulations with regard to the U.S. housing debt crisis are conducted with the consideration of the ZLB.

# B Sensitivity analysis

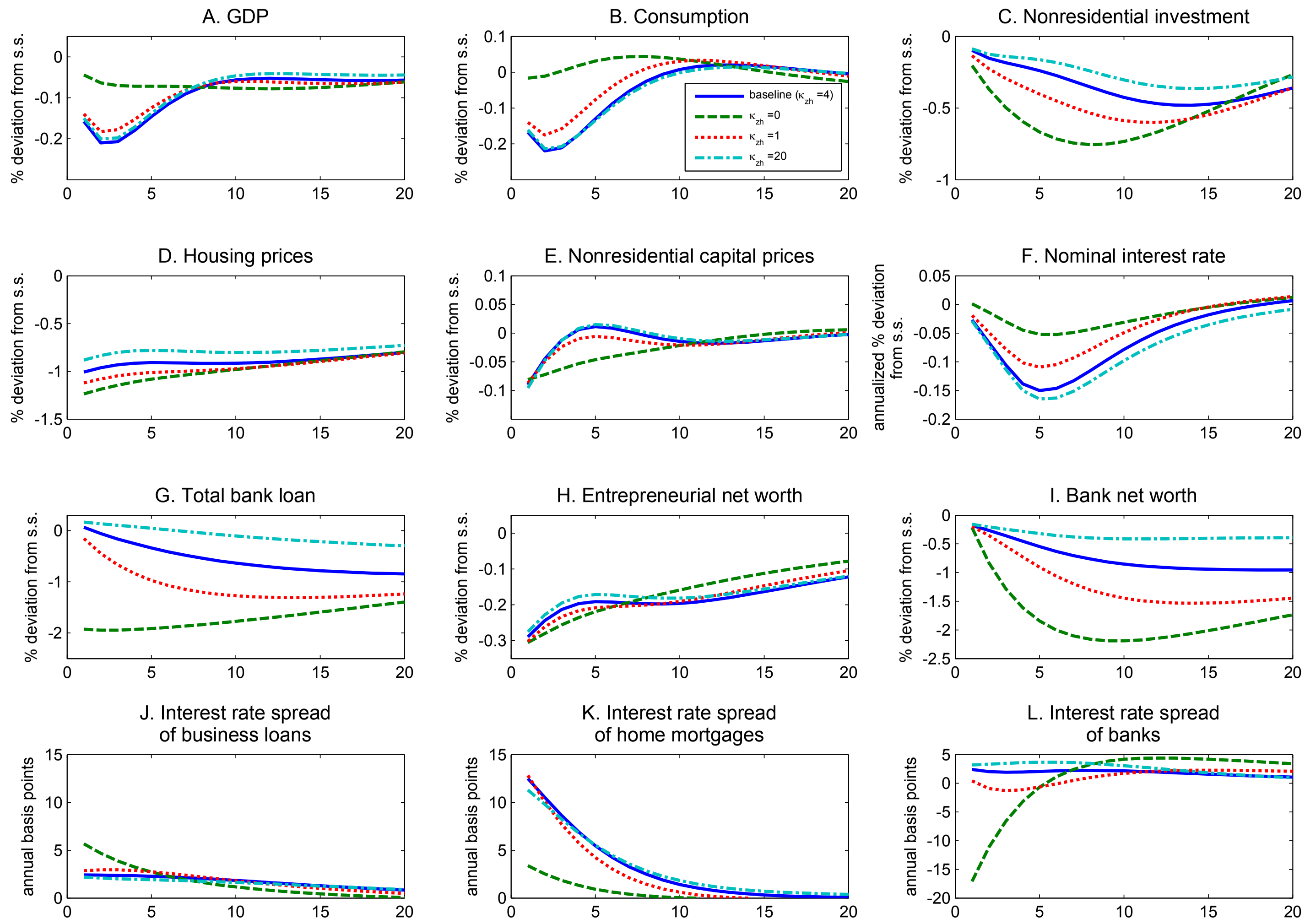
This appendix examines how the parameter for mortgage adjustment costs $\kappa_{zh}$ affects model dynamics. Figure B.1 presents impulse responses to a negative housing preference shock simulated from models with a different value for $\kappa_{zh}$. The simulation outcomes from the baseline model (solid blue lines) are the replication of the baseline case in Figure 2. That is, the size of the shock is chosen such that housing prices decrease on impact by 1% in the baseline model. Figure B.1 includes three other cases: a model with $\kappa_{zh} = 0$ (dashed green line), a model with $\kappa_{zh} = 1$ (dotted green line), and a model with $\kappa_{zh} = 20$[50] (dashed-dotted sky-blue line).

In the model with $\kappa_{zh} = 0$, where there is no cost to adjust home mortgages, home mortgages decline quickly in response to a negative housing demand shock. So does the housing stock. Therefore, given the size of the shock, housing prices drop by more than 1%. A bigger fall in housing prices depresses entrepreneurial net worth more and thus the demand for nonresidential capital diminishes further. In addition, entrepreneurs need to allocate more resources to take over real estate from impatient households. As a result, nonresidential investment decreases more than in the baseline case. The responses of consumption, however, are opposite. Due to a quick deleveraging of impatient households, the mortgage default rises a little, and in turn the lending rate spread of home mortgages increases marginally. The consumption of impatient households therefore decreases slightly. The aggregate consumption rather increases after the initial two-period decline, because patient households increase consumption in response to the reduction in policy rates. Meanwhile, the quick deleveraging of home mortgages causes the bank net worth to asset ratio to increase even in spite of larger losses on banks. It implies that the interest rate spread of banks declines and thus credit supply conditions improve. Due to this counterfactual accommodative reaction of banks, the output effects of a negative housing preference shock are much dampened compared to the baseline simulation.

The dynamic responses in the other two models ($\kappa_{zh} = 1$ & $\kappa_{zh} = 20$) are qualitatively same as the baseline model ($\kappa_{zh} = 4$). The greater the mortgage adjustment cost parameter is, the more sluggishly the variables related to banks move. Nevertheless, the output effects of a negative housing demand shock are almost same between the model with $\kappa_{zh} = 4$ and the model with $\kappa_{zh} = 20$.

[50]When the model with $\kappa_{zh} = 20$ is used for the simulation on the U.S. housing debt crisis (as in Section 4.2.1), the model-implied real total loans match well the empirical counterpart.

Figure B.1: IMPULSE RESPONSES TO A NEGATIVE HOUSING PREFERENCE SHOCK WITH DIFFERENT MORTGAGE ADJUSTMENT COST PARAMETERS




*Notes*: Horizontal axis represents quarters after the shock. The size of the shock is chosen such that housing prices decrease on impact by 1% in the baseline model. The shock process is identical across the models.

# C Equilibrium Conditions of the Baseline Model

In this section I summarize the equations describing the equilibrium of the baseline model.[51] The set of necessary conditions for an equilibrium is as follows.

**(Patient households)**

The real-term budget constraint of patient households:

$$C_t^P + Q_{h,t}[H_t^P - H_{t-1}^P] + D_t = R_{t-1}D_{t-1} + \frac{w_t^P L_t^P}{x_{w,t}^P} + \Upsilon_t^P, \tag{C.1}$$

where, $\Upsilon_t^P = \frac{x_{p,t}^P - 1}{x_{p,t}^P} Y_t + \frac{x_{w,t}^P - 1}{x_{w,t}^P} w_t^P L_t^P + \delta_b N_{t-1} + (1-\gamma)(1-\Gamma(\bar{\omega}_{e,t}))(R_t^k Q_{k,t-1} K_{t-1} + R_t^h Q_{h,t-1} H_{t-1}^E) - W^e + Tr_t^P$.

Optimality conditions:

$$\Lambda_t^P = \Gamma^P[\frac{1}{C_t^P - \varepsilon C_{t-1}^P} - E_t \frac{\beta^P \varepsilon}{C_{t+1}^P - \varepsilon C_t^P}], \tag{C.2}$$

$$\Lambda_t^P = \beta^P R_t E_t \Lambda_{t+1}^P, \tag{C.3}$$

$$(L_t^P)^{\varphi} = \frac{w_t^P}{x_{w,t}^P} \Lambda_t^P, \tag{C.4}$$

$$\Lambda_t^P Q_{h,t} = \frac{\chi_t^P}{H_t^P} + \beta^P E_t\{\Lambda_{t+1}^P Q_{h,t+1}\}. \tag{C.5}$$

**(Impatient households)**

The real-term budget constraint of impatient households:

$$C_t^I + Q_{h,t} H_t^I + ac_{zh,t} - Z_t^I = \frac{w_t^I L_t^I}{x_{w,t}^I} + \int_0^{\infty} \max\{\omega_t Q_{h,t} H_{t-1}^I - R_{t-1}^L Z_{t-1}^I, 0\} dF_{t-1}(\omega_t) + \Upsilon_t^I,$$

where, $ac_{zh,t} = \frac{\kappa_{zh}}{2} \frac{(Z_t^I - Z_{t-1})^2}{Z^I}$, $\Upsilon_t^I = \frac{x_{w,t}^I - 1}{x_{w,t}^I} w_t^I L_t^I + Tr_t^I$.

The decision rule for a default threshold, $\bar{\omega}_t$:

$$\bar{\omega}_t = \frac{R_{t-1}^L Z_{t-1}^I}{Q_{h,t} H_{t-1}^I} = \frac{m_t}{\Delta_{h,t+1}}, \tag{C.6}$$

here, the LTV ratio is defined as $m_t = \frac{R_t^L Z_t^I}{Q_{h,t} H_t^I}$, and the average realized return on housing is given by $\Delta_{h,t} = \frac{Q_{h,t}}{Q_{h,t-1}}$.

[51] Here, I allow for the possibility to include two risk shocks, which are a shock on the standard deviation of an idiosyncratic shock with respect to home mortgages and business loans, respectively.

The variables related to $\bar{\omega}_t$ are defined as below:

$$\begin{aligned}
F(\bar{\omega}_t;\sigma_{\omega,t-1}) &= \int_0^{\bar{\omega}_t} dF(\omega_t^j;\sigma_{\omega,t-1}) = \int_0^{\bar{\omega}_t} \frac{1}{\omega\sigma_{\omega,t-1}\sqrt{2\pi}} e^{-\frac{(\ln\omega+\frac{1}{2}\sigma^2_{\omega,t-1})^2}{2\sigma^2_{\omega,t-1}}} d\omega, \\
G(\bar{\omega}_t;\sigma_{\omega,t-1}) &= \int_0^{\bar{\omega}_t} \omega_t^j dF(\omega_t^j;\sigma_{\omega,t-1}) = 1-\Phi(\frac{\frac{1}{2}\sigma^2_{\omega,t-1}-\ln\bar{\omega}_t}{\sigma_{\omega,t-1}}), \\
\Gamma(\bar{\omega}_t;\sigma_{\omega,t-1}) &= [1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t + G_{t-1}(\bar{\omega}_t),
\end{aligned}$$

where, $F(\bar{\omega}_t,\sigma_{\omega,t-1})$ is a foreclosure rate ,$G(\bar{\omega}_t,\sigma_{\omega,t-1})$ is a fraction of foreclosed houses, and $\Gamma(\bar{\omega}_t;\sigma_{\omega,t-1})$ is a share of housing values going to the banks.

Then, we rewrite the budget constraint using the above expressions:

$$C_t^I + Q_{h,t}H_t^I + ac_{zh,t} - Z_t^I = \frac{w_t^I L_t^I}{x_{w,t}^I} + [1-\Gamma_{t-1}(\bar{\omega}_t)]Q_{h,t}H_{t-1}^I + \Upsilon_t^I. \tag{C.7}$$

Ex-ante participation constraint of bank (in real terms):

$$E_t\{(\Gamma_t(\bar{\omega}_{t+1}) - \mu^B G_t(\bar{\omega}_{t+1}))\Delta_{h,t+1}\}Q_{h,t}H_t^I = R_t^r Z_t^I. \tag{C.8}$$

Optimality conditions:

$$\Lambda_t^I = \Gamma^I[\frac{1}{C_t^I - \varepsilon C_{t-1}^I} - E_t\frac{\beta^I\varepsilon}{C_{t+1}^I - \varepsilon C_t^I}], \tag{C.9}$$

$$\Lambda_t^I[1-\kappa_{zh}\frac{\triangle Z_t^I}{Z^I}] = \beta^I E_t\{\Lambda_{t+1}^I[(\frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu^B G_t'(\bar{\omega}_{t+1})})R_t^r - \kappa_{zh}\frac{\triangle Z_{t+1}^I}{Z^I}]\}, \tag{C.10}$$

$$\begin{aligned}
\Lambda_t^I Q_{h,t} &= \frac{\chi_t^I}{H_t^I} + E_t\{\beta^I\Lambda_{t+1}^I[(1-\Gamma_t(\bar{\omega}_{t+1})) \\
&\quad + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu^B G_t'(\bar{\omega}_{t+1})}(\Gamma_t(\bar{\omega}_{t+1}) - \mu^B G_t(\bar{\omega}_{t+1}))]Q_{h,t+1}\},
\end{aligned} \tag{C.11}$$

$$(L_t^I)^\varphi = \frac{w_t^I}{x_{w,t}^I}\Lambda_t^I, m \tag{C.12}$$

where, $\Gamma_t'(\bar{\omega}_{t+1}) = 1 - F_t(\bar{\omega}_{t+1})$.

**(Entrepreneurs)**

The balance sheet of the entrepreneur at the period $t$:

$$Q_{k,t}K_t + Q_{h,t}H_t^E = Z_t^E + N_t^E. \tag{C.13}$$

The return to nonresidential capital $K_t$ and residential capital $H_t^E$ are given by, respectively:

$$R_{t+1}^k = \frac{r_{k,t+1} + (1-\delta_k)Q_{k,t+1}}{Q_{k,t}}, \tag{C.14}$$

$$R_{t+1}^h = \frac{r_{h,t+1} + Q_{h,t+1}}{Q_{h,t}}. \tag{C.15}$$

The decision rule for a default threshold, $\bar{\omega}_{e,t}$:

$$\bar{\omega}_{e,t} = \frac{R_{t-1}^E Z_{t-1}^E}{R_t^k Q_{k,t-1} K_{t-1} + R_t^h Q_{h,t-1} H_{t-1}^E}. \tag{C.16}$$

The variables related to $\bar{\omega}_{e,t}$ are defined as below:

$$\begin{aligned}
F(\bar{\omega}_{e,t};\sigma_{e\omega,t-1}) &= \int_0^{\bar{\omega}_{e,t}} dF(\omega_{e,t}^j;\sigma_{e\omega,t-1}) = \int_0^{\bar{\omega}_{e,t}} \frac{1}{\omega\sigma_{e\omega,t-1}\sqrt{2\pi}} e^{-\frac{(\ln\omega_e + \frac{1}{2}\sigma_{e\omega,t-1}^2)^2}{2\sigma_{e\omega,t-1}^2}} d\omega_e, \\
G(\bar{\omega}_{e,t};\sigma_{e\omega,t-1}) &= \int_0^{\bar{\omega}_{e,t}} \omega_{e,t}^j dF(\omega_{e,t}^j;\sigma_{e,\omega}) = 1 - \Phi(\frac{\frac{1}{2}\sigma_{e,\omega}^2 - \ln\bar{\omega}_{e,t}}{\sigma_{e,\omega}}), \\
\Gamma(\bar{\omega}_{e,t};\sigma_{e\omega,t-1}) &= [1 - F(\bar{\omega}_{e,t})]\bar{\omega}_{e,t} + G(\bar{\omega}_{e,t}),
\end{aligned}$$

where, $F(\bar{\omega}_{e,t},\sigma_{e,\omega})$ is a default rate , $G(\bar{\omega}_{e,t},\sigma_{e,\omega})$ is a fraction of the defaulted value of composite capital, and $\Gamma(e\bar{,}\omega_t;\sigma_{e,\omega})$ is the gross return of housing investment going to the banks.

The ex-ante participation constraint of the bank:

$$E_t\{(1 - F(\bar{\omega}_{e,t+1}))R_t^E Z_t^E + (1-\mu^E)\int_0^{\bar{\omega}_{e,t+1}} (R_{t+1}^k Q_{k,t} K_t + R_{t+1}^h Q_{h,t} H_t^E)\omega_{e,t} f(\omega_e) d\omega_e\} = R_t^r Z_t^E.$$

Two variables for the entrepreneurial leverage:

$$\phi_t^k = \frac{Q_{k,t} K_t}{N_t^E}, \tag{C.17}$$

$$\phi_t^h = \frac{Q_{h,t} H_t^E}{N_t^E}. \tag{C.18}$$

The bank's participation constraint can be reformulated using (C.13), (C.16), (C.17) and (C.18).

$$E_t\{[\Gamma(\bar{\omega}_{e,t+1}) - \mu^E G(\bar{\omega}_{e,t+1})](R_{t+1}^k \phi_t^k + R_{t+1}^h \phi_t^h)\} = R_t^r(\phi_t^k + \phi_t^h - 1). \tag{C.19}$$

The first order conditions are given by:

$$E_t\{(1 - \Gamma(\bar{\omega}_{e,t+1}))\frac{R_{t+1}^k}{R_t^r} + \frac{\Gamma'(\bar{\omega}_{e,t+1})}{\Gamma'(\bar{\omega}_{e,t+1}) - \mu^E G'(\bar{\omega}_{e,t+1})}[(\Gamma(\bar{\omega}_{e,t+1}) - \mu^E G(\bar{\omega}_{e,t+1}))\frac{R_{t+1}^k}{R_t^r} - 1]\} = 0, \tag{C.20}$$

$$E_t\{(1 - \Gamma(\bar{\omega}_{e,t+1}))\frac{R_{t+1}^h}{R_t^r} + \frac{\Gamma'(\bar{\omega}_{e,t+1})}{\Gamma'(\bar{\omega}_{e,t+1}) - \mu^E G'(\bar{\omega}_{e,t+1})}[(\Gamma(\bar{\omega}_{e,t+1}) - \mu^E G(\bar{\omega}_{e,t+1}))\frac{R_{t+1}^h}{R_t^r} - 1]\} = 0, \tag{C.21}$$

where, $w_t^k = \frac{\phi_t^k}{\phi_t^k + \phi_t^h}$.

Aggregate entrepreneurs' net worth evolves as follows:

$$N_t^E = \gamma[(1-\Gamma(\bar{\omega}_{e,t}))(R_t^k \phi_{t-1}^k + R_t^h \phi_{t-1}^h)\}N_{t-1}^E] + W^e. \tag{C.22}$$

**(Banking sector)**

The balance sheet of the bank at the period $t$:

$$\underbrace{Z_t^I + Z_t^E}_{W_t} = N_t + D_t. \tag{C.23}$$

Bank net worth is accumulated out of retained earnings:

$$N_t = (1-\delta_b)N_{t-1} + \Pi_{wb,t}, \tag{C.24}$$

where, retained earnings is given by $\Pi_{wb,t} = R_{t-1}^r W_{t-1} + \Pi_{hb,t} + \Pi_{eb,t} - R_{t-1}D_{t-1} - N_{t-1} - \kappa_w W_{t-1} - \frac{\phi_n}{2}(\frac{N_{t-1}}{W_{t-1}} - \nu_b)^2 N_{t-1}$.

The profit of each retail branch:

$$\begin{aligned} \Pi_{hb,t} &= \rho_h \Pi_{hb,t-1} + \underbrace{(\Gamma_{t-1}(\bar{\omega}_t) - \mu^B G_{t-1}(\bar{\omega}_t))Q_{h,t}H_{t-1}^I - R_{t-1}^r Z_{t-1}^I}_{\varepsilon_{hb,t}}, &\text{(C.25)} \\ \Pi_{eb,t} &= \rho_e \Pi_{eb,t-1} + \\ &\quad \underbrace{(\Gamma(\bar{\omega}_{e,t}) - \mu^E G(\bar{\omega}_{e,t}))(R_t^k Q_{k,t-1}K_{t-1} + R_t^h Q_{h,t-1}H_{t-1}^E) - R_{t-1}^r Z_{t-1}^E}_{\varepsilon_{eb,t}}. &\text{(C.26)} \end{aligned}$$

The first order condition of the wholesale branch:

$$R_t^r - R_t = \kappa_w - \phi_n(\frac{N_t}{W_t} - \nu_b)(\frac{N_t}{W_t})^2. \tag{C.27}$$

**(Production sector)**

The aggregate production technology:

$$Y_t = [H_{t-1}^E{}^{\nu_k} K_{t-1}{}^{1-\nu_k}]^{\alpha}[L_t^{P\nu_l} L_t^{I(1-\nu_l)}]^{(1-\alpha)}. \tag{C.28}$$

Optimality conditions from cost minimization:

$$\nu_l(1-\alpha)\frac{Y_t}{L_t^P} = x_{p,t}w_t^P, \tag{C.29}$$

$$(1-\nu_l)(1-\alpha)\frac{Y_t}{L_t^I} = x_{p,t}w_t^I, \tag{C.30}$$

$$(1-\nu_k)\alpha\frac{Y_t}{K_{t-1}} = x_{p,t}r_{k,t}, \tag{C.31}$$

$$\nu_k\alpha\frac{Y_t}{H_{t-1}^E} = x_{p,t}r_{h,t}. \tag{C.32}$$

Phillips curve:

$$\log(\frac{\Pi_t}{\Pi}) = \beta^P E_t \log(\frac{\Pi_{t+1}}{\Pi}) - \kappa_\pi \log(\frac{x_{p,t}}{x_p}), \tag{C.33}$$

where, $\kappa_\pi = \frac{(1-\theta_\pi)(1-\beta^P\theta_\pi)}{\theta_\pi}$.

The optimality condition of a capital goods producing firm:

$$Q_{k,t} = 1 + s_k(\frac{I_t}{I_{t-1}}) + s_k{}'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}} - E_t[\Lambda_{t,t+1}s_k{}'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2]. \tag{C.34}$$

Law of motion for capital:

$$K_{t+1} = (1-\delta_k)K_t + I_t. \tag{C.35}$$

**(Wage Phillips curves and the Government)**

Wage Phillips curves for the patient and impatient households:

$$\log(\frac{\Pi_{w^P,t}}{\Pi}) = \beta^P E_t \log(\frac{\Pi_{w^P,t+1}}{\Pi}) - \kappa_w^P \log(\frac{x_{w,t}^P}{x_w^P}), \tag{C.36}$$

$$\log(\frac{\Pi_{w^I,t}}{\Pi}) = \beta^I E_t \log(\frac{\Pi_{w^I,t+1}}{\Pi}) - \kappa_w^I \log(\frac{x_{w,t}^I}{x_w^I}), \tag{C.37}$$

where, $\Pi_{w^P,t} = \frac{w_t^P\Pi_t}{w_{t-1}^P}$, $\Pi_{w^I,t} = \frac{w_t^I\Pi_t}{w_{t-1}^I}$, $\kappa_w^P = \frac{(1-\theta_{w^P})(1-\beta^P\theta_{w^P})}{\theta_{w^P}}$, and $\kappa_w^I = \frac{(1-\theta_{w^I})(1-\beta^I\theta_{w^I})}{\theta_{w^I}}$.

Monetary policy:

$$R_t^n = \max\{1, (R_{t-1}^n)^{\gamma_R}(\frac{\Pi_t^A}{\Pi^A})^{(1-\gamma_R)\gamma_\pi}(\frac{Y_t}{Y})^{(1-\gamma_R)\gamma_y}(R^n)^{1-\gamma_R}\}, \tag{C.38}$$

where, $\Pi_t^A$ is year-on-year inflation (expressed in quarterly terms) and is defined as $\Pi_t^A = (P_t/P_{t-4})^{1/4}$.

The Fisher equation:

$$R_t = R_t^n / E_t\Pi_{t+1}. \tag{C.39}$$

Government budget constraint:

$$Tr_t^P + Tr_t^I = 0. \tag{C.40}$$

**(Market clearing and exogenous shocks)**

Housing market clearing:

$$H_t^P + H_t^I + H_t^E = 1. \tag{C.41}$$

By Walras' law, the good's market clears:

$$Y_t = C_t^P + C_t^I + [1 + s_k(\frac{I_t}{I_{t-1}})]I_t + Adj_t,$$

where, $Adj_t = \mu^B G_{t-1}(\bar{\omega}_t)Q_{h,t}H_{t-1}^I + \mu^E G(\bar{\omega}_{e,t})(R_t^k Q_{k,t-1}K_{t-1} + R_t^h Q_{h,t-1}H_{t-1}^E) + \kappa_w W_t + \frac{\phi_n}{2}(\frac{N_t}{W_t} - \nu_b)^2 N_t + ac_{zh,t}$.

Exogenous shock processes:

$$\begin{aligned}
\log(\chi_t^P) &= (1-\rho_\chi)\log(\chi^P) + \rho_\chi \log(\chi_{t-1}^P) + \varepsilon_{\chi,t}, & \text{(C.42)}\\
\log(\chi_t^I) &= (1-\rho_\chi)\log(\chi^I) + \rho_\chi \log(\chi_{t-1}^I) + \varepsilon_{\chi,t}, & \text{(C.43)}\\
\log(\sigma_{\omega,t}) &= (1-\rho_{\sigma_\omega})\log(\bar{\sigma}_\omega) + \rho_{\sigma_\omega} \log(\sigma_{\omega,t-1}) + \varepsilon_{\omega,t}, & \text{(C.44)}\\
\log(\sigma_{e\omega,t}) &= (1-\rho_{\sigma_{e\omega}})\log(\bar{\sigma}_{e\omega}) + \rho_{\sigma_{e\omega}} \log(\sigma_{e\omega,t-1}) + \varepsilon_{e\omega,t} & \text{(C.45)}
\end{aligned}$$

# D A modified equations for each policy experiment

The government earns funds by levying lump-sum taxes worth 1% of the steady-state annual GDP on patient households and makes a one-time lump-sum transfer to either banks or impatient households. I consider three cases below: a transfer to the banking sector, a transfer to impatient households, and a transfer to reduce the debt of impatient households.

$$Tr_t^P = -0.04Y^{GDP} \tag{D.1}$$

where, $Y^{GDP}$ is the steady state quarterly GDP.

## D.1 Lump-sum transfer to the banking sector: Bank capital injection

$-Tr_t^P$ is simply added to the equation for bank capital accumulations at period $t$.

$$N_t = (1-\delta_b)N_{t-1} + \Pi_{wb,t} - \mathbf{Tr_t^P} \tag{D.2}$$

## D.2 Lump-sum transfer to impatient households

$Tr_t^I$ is added to the budget constraint of impatient households.

$$\mathbf{Tr_t^I} = 0.04Y^{GDP} \tag{D.3}$$

## D.3 The reduction in the debt of impatient households

**(Impatient households)**

I firstly determine what fraction of the outstanding debt obligation is reduced with the funds of $0.04Y^{GDP}$. The fraction $\tau_t^r$ is an instrument for a debt relief policy.

$$Tr_t^I = 0.04Y^{GDP} = \boldsymbol{\tau_t^r} R_{t-1}^L Z_{t-1}^I \tag{D.4}$$

The budget constraint of impatient households is written with the instrument for the debt relief policy, $\tau_t^r$:

$$C_t^I + Q_{h,t}H_t^I + ac_{zh,t} - Z_t^I = w_t^I L_t^I + \int_0^\infty \max\{\omega_t Q_{h,t} H_{t-1}^I - (1-\boldsymbol{\tau_t^r}) R_{t-1}^L Z_{t-1}^I, 0\} dF(\omega_t) + \Upsilon_t^I \tag{D.5}$$

The default threshold, $\bar{\omega}_t$ will be lowered by the factor of $\tau_t^r$.

$$\bar{\omega}_t = \frac{(1-\boldsymbol{\tau_t^r}) R_{t-1}^L Z_{t-1}^I}{Q_{h,t} H_{t-1}^I} \tag{D.6}$$

As long as the policy is unexpected, other optimality conditions of impatient households do not change.

**(Banking sector)**

From the bank's perspective, the debt relief policy does not change the size of the bank's balance sheet, but alter the profit function of the retail branch for home mortgages. We can think this policy experiment as the case where the government transforms household debt into government debt and levies lump-sum taxes on patient households to repay debt with its interest. The bank's balance sheet at period $t$ after the government policy action:

$$(1-\boldsymbol{\tau}_{\mathbf{t}}^{\mathbf{r}})Z_{t-1}^{I} + \underbrace{Z_t^G}_{\boldsymbol{\tau}_{\mathbf{t}}^{\mathbf{r}} Z_{t-1}^{I}} + Z_{t-1}^{E} = Z_{t-1}^{I} + Z_{t-1}^{E} = N_{t-1} + D_{t-1}$$

The profit of the home mortgage branch:

$$\Pi_{hb,t} = \rho_h \Pi_{hb,t-1} + \underbrace{(\Gamma(\bar{\omega}_t) - \mu^B G(\bar{\omega}_t))Q_{h,t}H_{t-1}^{I} - R_{t-1}^{r}(1-\boldsymbol{\tau}_{\mathbf{t}}^{\mathbf{r}})Z_{t-1}^{I}}_{\varepsilon_{hb,t}} \tag{D.7}$$

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