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Abstract

I have assessed changes in the monetary policy stance in the euro area since its inception by applying a Bayesian time-varying parameter framework in conjunction with the Hamiltonian Monte Carlo algorithm. I find that the estimated policy response has varied considerably over time. Most of the results suggest that the response weakened after the onset of the financial crisis and while quantitative measures were still in place, although there are also indications that the weakening of the response to the expected inflation gap may have been less pronounced. I also find that the policy response has become more forceful over the course of the recent sharp rise in inflation. Furthermore, it is essential to model the stochastic volatility relating to deviations from the policy rule as it materially influences the results.

Keywords: Monetary policy rules, Bayesian time-varying parameter estimation, unconventional monetary policy, Hamiltonian Monte Carlo
JEL classification: E52, C11, C22, C51

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1. Introduction

Simple policy rules have often been applied to assess the stance of monetary policy since the seminal contribution made by Taylor (1993). Several studies exist – mainly for the US – which estimate simple rules using regime-switching models or time-varying parameter frameworks to find evidence of changes in monetary policy over time (e.g. Sims (2001), Cogley and Sargent (2001, 2005), Boivin (2006), Kim and Nelson (2006), Sims and Zha (2006)).[1] However, the latter strand of the literature appears to be limited for the euro area.

In this paper I expand the available literature by estimating the evolution of the monetary policy response in the euro area since the inception of monetary union. I apply a Bayesian non-linear framework in combination with the Hamiltonian Monte Carlo (HMC) algorithm, which is superior to other existing sampling methods owing to its efficiency and unique diagnostic features. This modelling framework allows for both time-varying parameters and heteroscedasticity within a euro-area-specific monetary policy rule and does not require any linear approximation or the application of the Kalman filter. I scrutinise to what extent the non-standard monetary policy measures have influenced the policy response in the aftermath of the financial crisis and attempt to shed light on the role that monetary policy may have played in the run-up to the recent sharp rise in inflation in the euro area. Furthermore, I evaluate the European Central Bank's (ECB) recent policy response to the surge in inflation.

A remarkable amount of earlier literature provides point estimates mostly of Taylor-type or ad-hoc reaction functions of the ECB.[2] Gorter et al. (2010) applied a rolling-window OLS approach to assess possible changes in the ECB's reaction function during the financial crisis, while Gerlach and Lewis (2014a, 2014b) used a smooth transition framework to allow for shifts between two different parameter sets for the estimated reaction function. Rostagno et al. (2019) estimated policy rules for

[1] Earlier studies such as e.g. Judd et al. (1998), Clarida et al. (2000) or Orphanides (2004) used subsample analysis for this purpose.

[2] See, for example, the studies cited in Blattner and Margaritov (2010) or Gerlach and Lewis (2014b).

the period from 1999 to 2008 and found a more forceful response when inflation was above its 2 percent target than when it was below this level. Maih et al. (2021) estimate, among other things, a regime-switching version of the Smets-Wouters DSGE model up to the end of 2014.2. They allow for an endogenous switch of the coefficient on inflation within a smoothed Taylor-type policy rule depending on whether inflation is above or below its target. They conclude that until mid-2014 the ECB responded more forcefully to inflation of above 1.9 percent than it did to inflation below that level. Furthermore, based on their estimation results up to mid-2014, they show by likelihood comparison that the ECB conducted a symmetric policy during the period after mid-2014. One drawback with this approach is that it does not enable the policy response to be directly estimated while quantitative easing was in place. In addition, only the switch in the ECB's response to inflation is taken into account, although its response to other economic aggregates within the policy rule may also have changed, which would probably alter the relevant results. Furthermore, this methodology does not allow continuously changing responses during the assessed period to be estimated. Paloviita et al. (2021) use GMM and a rolling-window approach to estimate a large variety of Taylor-type policy rules for the period from 1999 to 2014. They conclude that policy responses have been asymmetric, with the responses to inflation overshooting its target being more forceful than those to inflation undershooting.

This paper departs from the framework applied in Orphanides and Wieland (2013) (OW henceforth) and Bletzinger and Wieland (2017) (BW henceforth). They characterise the ECB's reaction function by assuming that the policy rate is set according to the following rule:

$$i_t = i_{t-1} + \theta_1(\pi_{t+3|t} - \pi^*) + \theta_2(q_{t+2|t} - q^*_{t+2|t}) \tag{1}$$

Here, i denotes the policy rate, π the inflation rate, π^* the inflation target, q the growth rate of real GDP and q^* the growth rate of real potential GDP. The subscript $t+i|t$ refers to the expected value at time t of a particular variable i quarters ahead.

A change in the policy rate compared with the level in the previous period occurs when expected inflation deviates from the central bank's inflation target or expected GDP growth deviates from expected potential growth. OW show that by setting the parameters θ_1 and θ_2 to 0.5 each and the inflation target to 1.5 or 2.0, the policy rate prescriptions under the policy rule using data from the Survey of Professional Forecasters (SPF) and the European Commission are very similar to the policy rate set by the ECB over time. This rule for the euro area was estimated by, among others, Smets (2010), BW and Hartmann and Smets (2018) (HS henceforth), who applied the following framework:[3]

$$\Delta i_t = c + \theta_1 \pi_{t+3|t} + \theta_2 (q_{t+2|t} - q^*_{t+2|t}) + \epsilon_t \tag{2}$$

where Δi_t is the change in the policy rate compared with the previous period, c is a constant and ϵ_t denotes the error term with $\epsilon_t \sim N(0, \sigma^2)$. Consequently, the estimated inflation target is given by $\pi^* = c/\theta_1$. BW estimated θ_1 and θ_2 at 0.49 and 0.40, respectively, and the inflation target at 1.72 in their baseline specification. HS estimated the above parameters with staff projections of the ECB/Eurosystem instead of SPF forecasts for the period from 1999.1 to 2018.1 in their baseline specification at 0.34, 0.37 and 1.81, respectively. To assess the stability of the parameter estimates over time HS carried out the estimation with a shortened sample until 2012.2. They report that the estimation results differ only slightly from the estimates based on the full sample suggesting that the regression coefficients should remain stable over time.

The latter studies have major shortcomings, though. Firstly, they partly disregard that the reaction function of a central bank can vary over time. Even if the regression is estimated on a shorter sample as in HS, the estimated inflation target will change as well, although the monetary policy strategy remained the same over the entire sample period. This will possibly bias the estimates of the reaction coefficients. Secondly, these studies do not take account of the effect that quantitative

[3] Smets (2010) regressed the interest rate on its lag but estimated fairly high values from 0.89 (baseline setup) up to 0.97, depending on the model specification.

easing has on the key policy rate. Although HS, for example, used a time series for their estimation which combined the changes in the MRO up to 2008.3 with the changes in the deposit rate from 2008.4 onwards, the latter series may not fully capture the magnitude of the non-standard measures adopted by the ECB in the aftermath of the financial crisis. Thirdly, none of these studies applied a time-varying parameter framework, which would provide accurate estimates of the central bank's response in the euro area over time and may therefore result in wrong inference if comparing different time periods.

In this paper I attempt to remedy these shortcomings by providing a consistent framework for measuring continuous changes in the monetary policy response up to the present. I find that the estimated policy response has varied considerably over time. In particular, most of the results suggest that the policy response weakened after the beginning of the financial crisis and while quantitative measures were in place. There are also indications, however, that the weakening of the response to the expected inflation gap may have been less pronounced, depending on the data set used for estimation purposes. I also find that the policy response has strengthened during the recent surge in inflation. Moreover, the introduction of stochastic volatility associated with deviations from the policy rule is also essential, as it materially affects the results.

The rest of this paper is organized as follows. Sections 2 describes the model setup and the estimation approach. Section 3 provides a description of the data. Section 4 discusses the baseline results. Section 5 presents robustness checks and extensions of the baseline framework. Section 6 concludes the paper.

2. A Bayesian time-varying parameter representation

To describe the law of motion of the ECB's response to macroeconomic aggregates I apply a Bayesian non-linear regression framework with time-varying parameters which is well suited to capture the gradual change in the response of monetary policy over time. I estimate the policy rule from OW, similarly to BW, but treat the reaction coefficients $\theta_{1,t}$ and $\theta_{2,t}$ as being random to obtain the posterior densities of both coefficients for all points in time t in the period from 1999.2 to 2021.2 for the euro area.

2.1. The model

The model can be represented by means of a non-linear state space where the observation equation is given by

$$y_t = f(X_t, \tilde{\theta}_t) + \epsilon_t \qquad t = 1, ..., T. \tag{3}$$

y_t is a scalar variable, $\tilde{\theta}_t$ is a $K \times 1$ dimensional vector of parameters to be estimated, X_t is a vector of predetermined or exogenous variables and ϵ_t is the error term. For this case, y_t is set to be the change in the key policy rate at time t, Δi_t, the left-hand side of the policy rule in OW. The time-varying coefficients are treated as the hidden state vector in the state space. To describe the dynamics of the reaction coefficients, θ_1 and θ_2, over time, I postulate a driftless random walk process:

$$\theta_{i,t} = \theta_{i,t-1} + \nu_{i,t} \qquad i = 1, 2 \tag{4}$$

where I assume that $\nu_t' := (\nu_{1,t}, \nu_{2,t})$ is i.i.d. with a covariance matrix Q. This allows to characterize the evolution of $\theta_t := (\theta_{1,t}, \theta_{2,t})$ over time as follows:

$$p(\theta_{t+1}|\theta_t, \Omega) \propto h(\theta_{t+1}|\theta_t, \Omega) \tag{5}$$

where $p(\theta_{t+1}|\theta_t, \Omega)$ is the conditional probability density of the state vector for the next period, θ_{t+1}, given the state vector in the current period, θ_t, as well as Ω, denoting the covariance matrix of the state space to be defined below and

$$h(\theta_{t+1}|\theta_t, \Omega) \sim N(\theta_t, Q). \tag{6}$$

Furthermore, I also assume that the error terms, $(\epsilon_t, \nu_t')'$, are i.i.d. normal random variables with zero mean and a covariance matrix

$$\Omega := \begin{bmatrix} \epsilon_t \\ \nu_t \end{bmatrix} \begin{bmatrix} \epsilon_t & \nu_t' \end{bmatrix} = \begin{bmatrix} \sigma^2 & C' \\ C & Q \end{bmatrix} \tag{7}$$

where σ^2 is the variance of ϵ. Q is the covariance matrix of the innovations of the state equation and C denotes the cross covariances. To derive the joint likelihood function let us first define $\tilde{\theta}_t := (\theta_t, \pi^*)$, the vector consisting of the states at time t and the inflation target. In the following I will denote the partial histories of the states and the observed variable by $\theta^t := [\theta_1, ..., \theta_t]$ and $Y^t := [y_1, ..., y_t]$, respectively, for all $t = 1, ..., T$. Consequently, the partial history of $\tilde{\theta}$ is defined as $\tilde{\theta}^t := [\theta_1, ..., \theta_t, \pi^*]$. To derive the joint posterior density function one can exploit first the probabilistic structure of the random walk process characterizing the time-varying parameters:

$$p(\theta^T|\Omega) = p(\theta_0|\Omega) \prod_{t=0}^{T-1} p(\theta_{t+1}|\theta_t, \Omega). \tag{8}$$

According to Bayes' law, the posterior density can be expressed as the product of the likelihood and the joint prior:

$$p(\tilde{\theta}^T, \Omega|Y^T) \propto p(Y^T|\tilde{\theta}^T, \Omega)p(\tilde{\theta}^T, \Omega). \tag{9}$$

Since the observations are conditionally independent and by assuming that the cross covariance is zero, the likelihood can be factored as follows:

$$p(\tilde{\theta}^T, \Omega|Y^T) \propto p(\tilde{\theta}^T, \Omega) \prod_{t=0}^{T} p(y_t|\tilde{\theta}^T, \Omega) = p(\tilde{\theta}^T, \Omega) \prod_{t=0}^{T} p(y_t|\tilde{\theta}^t, \Omega). \tag{10}$$

The equation holds because the policy rate setting rule does not depend on future values of the reaction coefficients. Furthermore, given the probabilistic structure of θ stated above and since the central bank changes the policy rate only in response to the expected inflation gap and the expected growth gap, the posterior can be expressed as follows:

$$p(\tilde{\theta}^T, \Omega | Y^T) \propto p(\tilde{\theta}^T, \Omega) \prod_{t=0}^{T} p(y_t | \tilde{\theta}_t, \Omega). \tag{11}$$

Following Koop (2003) the likelihood function of the non-linear univariate regression model can be written in vector form:

$$p(y|\tilde{\theta}, \sigma) = \frac{1}{(2\pi\sigma^2)^{N/2}} \left\{ \exp\left[-\frac{1}{2\sigma^2} \{y - f(X, \tilde{\theta})\}' \{y - f(X, \tilde{\theta})\} \right] \right\}. \tag{12}$$

Setting $f(X, \tilde{\theta}) = \theta_1(\pi^e - \pi^*) + \theta_2(q^e - q^{*,e})$ the likelihood will translate into

$$p(Y^T | \tilde{\theta}^T, \Omega) = \prod_{t=0}^{T} \frac{1}{(2\pi\sigma^2)^{1/2}} \left\{ \exp\left[-\frac{(y_t - f(X_t, \tilde{\theta}_t))^2}{2\sigma^2} \right] \right\} \tag{13}$$

$$\text{with} \quad f(X_t, \tilde{\theta}_t) := \theta_{1,t}(\pi_{t+3|t} - \pi^*) + \theta_{2,t}(q_{t+2|t} - q^*_{t+2|t}). \tag{14}$$

The joint prior can be further rewritten by factoring it into the conditional prior for θ^T and the joint density of the inflation target and the state space covariance:

$$p(\tilde{\theta}^T, \Omega) \equiv p(\theta^T, \pi^*, \Omega) = p(\theta^T | \pi^*, \Omega) p(\pi^*, \Omega). \tag{15}$$

I assume that the central bank response is characterized by a random walk process which does not depend on the inflation target. This may appear slightly restrictive, however it could be very challenging to specify a relation and consequently a conditional distribution for θ depending on π^* as well, if there exists any at all. Therefore, given the assumption that the central bank response is independent from π^*, the latter can be dropped from the conditional distribution of θ^T, so one can make directly use of equation (8). Furthermore, assuming that the initial state, θ_0, and the covariance matrix of the state space are independent, the prior can be expressed as follows:

$$p(\theta^T | \pi^*, \Omega) p(\pi^*, \Omega) = p(\pi^*) p(\Omega) p(\theta_0) \prod_{t=0}^{T-1} p(\theta_{t+1} | \theta_t, \Omega). \tag{16}$$

To complete the model the prior distributions have to be specified. In the baseline setup, I assume a Gaussian distribution for the initial states:

$$p(\theta_0) \propto N(\bar{\theta}, \bar{P}) \tag{17}$$

with $\bar{\theta} = (0.5, 0.5)$ and a diagonal $\bar{P}$ matrix. I set a relatively wide prior with entries on the main diagonal of $\bar{P}$ being equal to $\bar{p}_{11} = \bar{p}_{22} = 4$ to let the data determine the initial value. The prior distribution of the inflation target is assumed to be $\pi^* \sim N(1.9, 0.15^2)$ which reflects the monetary policy strategy of the ECB, in particular that inflation is targeted to be "below, but close to 2 percent". For the prior distribution of the state space covariance matrix, Ω, I will assume that the cross covariance, C, is zero. Furthermore, the covariance matrix structure of the state equation innovations, Q, is assumed to be diagonal. Consequently, for the standard deviation of both random walk innovations, σ_{ν_1} and σ_{ν_2}, and for the standard deviation of the error term in the policy rule, σ, I will posit an inverse gamma distribution with an expected value of 0.2 and a relatively wide standard deviation of 0.2, respectively.[4] I will use the above priors to first estimate the model on the training sample between 1999.2 and 2001.4. Subsequently, the results obtained using the latter sample will be set as priors for the estimation of the complete sample.

2.2. Simulation of the posterior density

The estimation of the policy rule from (1) in its original form in conjunction with time-varying coefficients poses a serious challenge as the observation equation is non-linear. A straightforward approach to address non-linearity issues would be to follow the setup in BW, yet to estimate equation (2) with a time-varying intercept and to rely on established estimation methods. However, this would result in identification problems and disregard at the same time that the estimated intercept depends on θ_1. Furthermore, the implied inflation target would be time-varying as well. Assuming that θ_1 and θ_2 are time-varying requires the estimation of more than

[4] This corresponds to $\sim IG(\alpha, \beta)$ with $\alpha = 3$ and $\beta = 0.4$.

$2T$ parameters. A readily available Bayesian estimation method capable of sampling from high-dimensional distributions, given that a posterior distribution can be derived, is the HMC algorithm. The latter approach has been widely applied in other fields of science owing to its efficiency and superior diagnostic features, see e.g. Neal (2011) and Betancourt (2017). The main idea of the HMC algorithm is to extend the parameter space by an auxiliary vector of momentum variables, α, to obtain the joint posterior density of $\tilde{\theta}^T$, Ω and α:

$$p(\tilde{\theta}^T, \Omega, \alpha | Y^T) \propto p(Y^T | \tilde{\theta}^T, \Omega, \alpha) p(\tilde{\theta}^T, \Omega, \alpha). \tag{18}$$

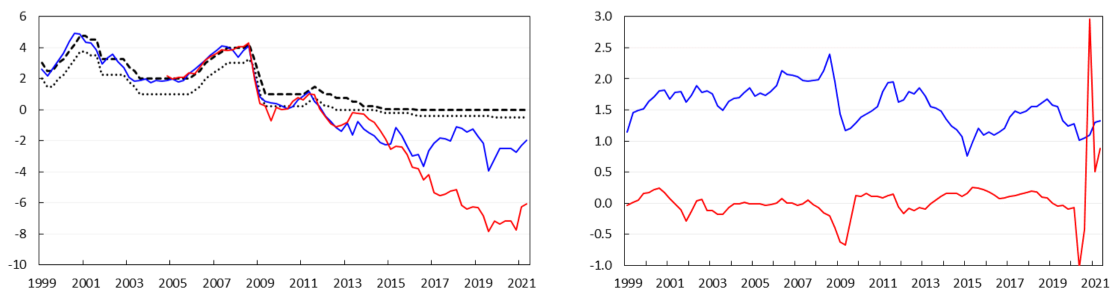
To each parameter to be estimated in the original model, one momentum variable α_i is assigned which is a priori independent of $\tilde{\theta}^T$, Ω and Y^T so that $p(\tilde{\theta}^T, \Omega, \alpha | Y^T) \propto p(Y^T | \tilde{\theta}^T, \Omega) p(\tilde{\theta}^T, \Omega) p(\alpha)$. In the extended parameter space one can apply Hamiltonian dynamics, a well known concept from physics. The Hamiltonian equation describes the total energy of a frictionless mechanical system, in particular the position and the momentum of a moving particle. In this extended framework, the model parameters to be estimated are considered as the position of a particle. In a frictionless mechanical system the Hamiltonian equation ensures that the energy is conserved. Consequently, the application of the Hamiltonian equation in this extended parameter space allows to find an update proposal which is distant from the original position of the particle, that is, of the parameters to be estimated, while the acceptance rate remains very high, even if the dimension of the parameter space is high. For further details I refer to Neal (2011) and Betancourt (2017) who provide an excellent description of the methodology along with its superior features compared to standard random walk based sampling methods.

3. Data description

I use a quarterly data set for the estimation which is partially compiled from monthly data. As regards the change in the key policy rate I use the first difference of the main refinancing operations (MRO) rate of the ECB until 2008.2. From

2008.3 onwards, when the global financial crisis started to affect the euro area, I replace the latter series with the first difference of the shadow interest rate series. Thereby I account for the impact of non-conventional measures in periods when the effective lower bound was possibly binding. I also remedy a further shortcoming in the existing literature which does not take at all, or at least not properly, account of the effects of the quantitative measures carried out by the ECB. Although HS uses the deposit rate instead of the key policy rate from 2008.4 onwards, the latter series reflects the changes in the policy rate only partly, at best, as can be seen in Figure 1. Two different monthly shadow rate series are regularly estimated and updated

Figure 1: Interest rates, expected inflation and growth gap in the euro area



(a) Policy Rates and Shadow Rates (b) Expected Inflation and Growth Gap

Notes: Plot (a): the dashed line shows the MRO rate and the dotted line the deposit facility rate of the ECB. The blue and the red lines represent the shadow interest rates based on the Krippner and the Wu-Xia datasets, respectively. Plot (b): the blue line shows the expected HICP inflation in $t+3$, while the red line represents the difference between the expected GDP growth and forecasted potential GDP growth in $t+2$.

by Krippner and Wu, respectively, based on their previous studies (Krippner (2013, 2015); Wu and Xia, (2017, 2020)). The series differ remarkably, therefore I will carry out all estimations with both series separately. In each case, also for the MRO, I use rates in the second month of the quarter to build the first differences, as the Survey of Professional Forecasters (SPF), where the expected HICP inflation and the expected GDP growth series stem from, are published at the end of the first month each quarter. Therefore, both explanatory variables could be considered as weakly exogenous. As in BW and HS, I use the 4-quarters-ahead forecasts from the most recent data point available for both explanatory variables. I use forecast data from the SPF, in particular, expected yearly HICP inflation in $t+3$ quarters and expected yearly GDP growth in $t+2$ quarters ahead owing to the different availability of the HICP and the GDP growth data series. To calculate the forecasted yearly potential growth rate on a quarterly basis I rely on the yearly estimates and projections of the European Commission as no publicly available estimates are provided by the ECB and transform the periodicity of the series by quadratic interpolation.[5]

Figure 1 shows that in normal times, when the central bank was not constrained by the zero lower bound and large scale asset purchases were not used as a standard policy instrument, the shadow rates differ only marginally from the MRO rate. However, with the onset of the financial crisis and the subsequent introduction of the non-standard measures the shadow rates fell significantly below the MRO rate. Although the gap between the shadow rates and the MRO rate closed in the first half of 2011, the subsequent measures taken to support both the private and the public sector in the euro area caused the shadow rates to diverge again significantly from the MRO rate. The gap widened even further with the introduction of the different asset purchase programmes until the end of 2016 when the pace of the asset purchases decreased. At this point, according to both data sets the shadow rate hovered around in negative territories well below the MRO rate and both shadow rate series showed a similar pattern. From the end of 2016 they exhibited a markedly different dynamics though. As a consequence of the subsequent reduction in net asset purchases the Krippner series has already signaled a tightening while according to the Wu-Xia series the shadow rate became even more negative. The different evolution continued until the termination of the asset purchase programmes at the end of 2018. From 2019.2 both series pointed again to an easing in accordance with the relaunch of the longer term refinancing operations and the restart of the asset purchases when inflation expectations started deteriorating again. Although at the beginning of the pandemic a further asset purchase programme was announced both shadow rate series fail to signal a monetary easing. This feature is possibly attributable to a substantial increase in the demand for liquidity due to the complete shutdown of

[5] I rely on the standard procedure provided in the software package Eviews.

the economy. In the first half of 2021 the shadow rates signaled again a tightening due to the repeated pandemic related shutdown of numerous economies in the euro area. Both the MRO and the deposit rate rate barely changed from 2015 onwards and clearly failed to reflect the impact of quantitative easing adequately. Therefore, the application of shadow rates for the estimation is warranted.

As regards expected inflation, it follows the same pattern as the realized inflation. It is less volatile, however, and significantly smoother than the realized series. Until the beginning of 2008 it remained close to two percent and shortly before the start of the financial crisis it picked up slightly. Following the relatively steep decline it returned again to two percent. After the outbreak of the sovereign debt crisis in the euro area expectations gradually declined until the beginning of 2015 to approximately 0.75 percent. In course of the pickup in economic activity inflation expectations gradually moved back again to slightly less than 1.7 percent at the end of 2018. With the slowdown of the economy and the subsequent outbreak of the pandemic it declined continuously until mid-2020 and has been increasing since then. The expected growth gap series follows the pattern of the realized series but exhibits a smooth dynamics. However, in general it captures periods of overheating and positive growth gap less well. Given its forward looking nature the spike in the growth gap occurs two periods earlier, as after the first wave of the pandemic with the reopening of the economy it became foreseeable that the economic activity will return to approximately previous levels.

4. Results

To estimate the model I applied Stan, a state-of-the-art, freely available software which implements the HMC algorithm in combination with the No-U-Turn (NUTS) sampler.[6] For the baseline estimation I trained the model with the priors from Section

[6]To summarize, the NUTS sampler ensures that in the extended space, in that Hamiltionian dynamics is applied to find an update proposal, the HMC algorithm stops at some point where the distance from the original point of departure does not increase anymore. Hence, the algorithm makes no u-turn which would lead to a waste of computational resources. For further details on the NUTS sampler I refer to Hoffman and Gelman (2014).

2 on the data sample from 1999.2 to 2001.4. The fine-tuning of the HMC/NUTS

Table 1: Priors and posterior estimates – training sample from 1999.2 to 2001.4

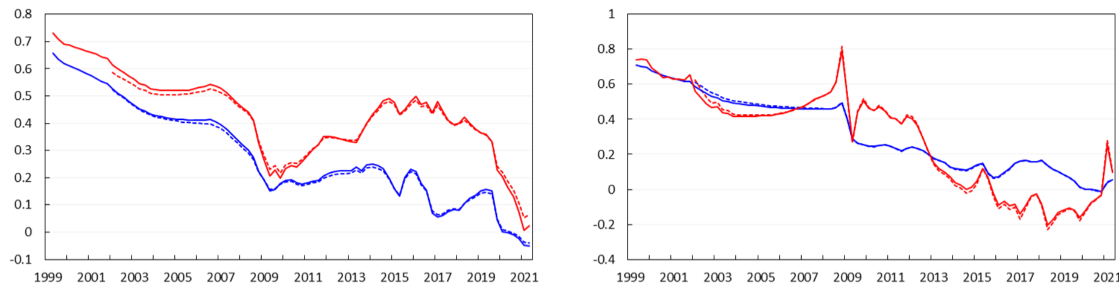
		Prior		Posterior		
Param.	Density	Mean	Std	Mean	Std	[0.05, 0.95]
π^*	Normal	1.90	0.15	1.87	0.11	[1.72, 2.07]
σ	InvGamma	0.20	0.20	0.20	0.056	[0.12, 0.30]
σ_{ν_1}	InvGamma	0.10	0.05	0.18	0.14	[0.063, 0.44]
σ_{ν_2}	InvGamma	0.10	0.05	0.16	0.094	[0.061, 0.34]
$\theta_{1,0}$	Normal	0.50	2.00	0.91	0.44	[0.26, 1.67]
$\theta_{2,0}$	Normal	0.50	2.00	0.89	0.37	[0.37, 1.54]
$\overline{\theta}_1$	–	–	–	0.90	0.55	[0.11, 1.84]
$\overline{\theta}_2$	–	–	–	0.76	0.22	[0.41, 1.14]

Notes: The table shows the prior distributions set for the estimation of the model on the training sample comprising the time period from 1999.2 to 2001.4 and the posterior means, standard deviations and the 90 percent credible intervals (in brackets) of the particular parameters. The values for $\overline{\theta}_1$ and $\overline{\theta}_2$ correspond to the average of the estimated posterior means and standard deviations of each individual $\theta_{1,t}$ and $\theta_{2,t}$ at time t over the period from 1999.2 to 2001.4.

sampler was carried out automatically by Stan based on a burn-in sample of 10,000 draws for each of the four chains. The estimates for the posterior distributions were obtained based on 50,000 sample draws per chain. The estimated posterior means and standard deviations for the training sample are summarized in Table 1. These posteriors were taken then as priors for the main estimation exercise making use of the complete data set until 2021.2.

For the period from 1999.2 to 2001.4 the posterior mean for the inflation target, π^*, is estimated at 1.87 percent which corresponds almost to its prior mean. Yet, the standard deviation is lower than initially assumed implying that the data is informative. The posterior means of the initial values of both response coefficients, $\theta_{1,0}$ and $\theta_{2,0}$ are considerably higher with approximately 0.9 than their prior means and significantly more narrow with estimated standard deviations of 0.44 and 0.37, respectively. The averages of the posterior estimates of each individual $\theta_{1,t}$ and $\theta_{2,t}$ at time t over the period from 1999.2 to 2001.4, denoted by $\overline{\theta}_1$ and $\overline{\theta}_2$, are estimated at 0.90 and 0.76. This implies that the response to the expected growth gap declined over the first three years. The conditional standard deviations of the random walk processes of the time-varying reaction coefficients, σ_{ν_1} and σ_{ν_2}, are lower with 0.18 and 0.16 than initially assumed and their posteriors are more tightly shaped. The estimation based on the complete sample was carried out again with four

Figure 2: Posterior estimates of time-varying reaction coefficients



(a) Expected Inflation Gap (b) Expected Growth Gap

Notes: Plot (a) shows the estimates for the time-varying response to the expected inflation gap, $\theta_{1,t}$, and plot (b) to the expected growth gap, $\theta_{2,t}$. The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate rate series, respectively. The dashed lines correspond to the estimates for the period from 2002.1 to 2021.2 and the solid lines to the period from 1999.2 to 2021.2. The priors are set according to the results in Table 1.

parallel chains, each with a burn-in of 10,000 draws to fine-tune the HMC/NUTS-sampler and subsequently with 50,000 sample draws. For the sake of completeness, I carried out the estimation both on the sample from 2002.1 to 2021.2 by excluding the training sample and on all available data from 1999.2 to 2021.2 as well, see Figure 2. To this respect, the results differ only marginally. Including the data in the sample from 1999.2 to 2001.4 as well results in a smooth continuation of the estimates. However, one needs to acknowledge that for the latter case the priors are not entirely valid as also future information is used. The intention was to showcase whether and to what extent the estimation results with respect to the future response are influenced by the initial period of a seemingly aggressive response by the ECB at the end of the 1990s when reputation had to be build up. The evolution of the response to the expected inflation gap is shown in plot (a) and to the expected growth gap in plot (b). The blue lines refer to the estimation results obtained by using shadow rates from 2008.3 onwards provided by Krippner while the red lines show the results when carrying out the estimation with the Wu-Xia shadow rate series.[7] In general, both reaction coefficients feature a declining trend over time. In particular, the average response of the ECB to expected deviations from the inflation target has declined from its relatively high initial values but stabilized then around 0.5. With the beginning of the financial crisis, the response of the ECB to the expected inflation gap has declined further though. Considering the estimation results based on the shadow rates provided by Krippner, the softening continued and became statistically significant towards the end of the sample period. By contrast, the results obtained with the Wu-Xia series suggest that the response to the expected inflation gap rebounded in the aftermath of the financial crisis and reached values in the range of 0.5. Following its local peak at the beginning of 2016 it has been declining at an increasing pace and reached almost as low values as the response obtained with the Krippner series. The difference in the parameter estimates can be explained by the steep negative progression of the Wu-Xia shadow rate series. The response to the expected growth gap has also declined since the financial crisis and converged to zero if estimated with the Krippner series. The estimation results obtained with the Wu-Xia series exhibit a similar pattern, yet the response is more volatile. After 2015 it reaches even negative values but turns out to be positive at the end of the sample period.

The posterior estimates for the time-invariant parameters and the initial values of the response to the expected inflation gap and the expected growth gap are shown in Table 2. The estimated posterior mean of the inflation target, π^*, is slightly lower than the estimate obtained using the training sample. Both estimates of π^* obtained with the two different shadow rate series, are almost similar, 1.83 if estimated with the Krippner series and 1.81 with the Wu-Xia series. Furthermore, the estimated posterior standard deviations are also lower with 0.09 and 0.08, respectively, than the estimates obtained with the training sample. The posterior mean of σ, the standard deviation of the remainder in the policy rule, is approximately twice as large if compared with the estimate based on the training sample. With the Krippner series

[7] Figure A.1 in the Appendix shows the results in separate charts including the 90 percent credible intervals.

it is estimated at 0.43 that is larger to some extent than the result obtained with the Wu-Xia series, namely 0.37. The posterior means of the conditional standard deviations of the time-varying response to the expected inflation gap, σ_{ν_1}, are lower than for the training sample and are fairly similar for the two different shadow rates series, estimated at 0.13 and 0.14, respectively. The standard deviations of the posterior distributions of σ_{ν_1} are considerably lower with 0.080 and 0.093. The posterior mean of the conditional standard deviation of the time-varying response to the expected growth gap, σ_{ν_2}, differs though for the two different data sets, estimated at 0.10 with the Krippner series and at 0.18 with the Wu-Xia series. Hence, the Wu-Xia series generates a more volatile response to the expected growth gap than the Krippner series. Furthermore, the posterior standard deviations of σ_{ν_2} for both shadow rate series, estimated at 0.035 and 0.084 respectively, are also lower than the estimates obtained with the training sample, especially the one for the Krippner series.

Table 2: Posterior estimates for the time period from 1999.2 to 2021.2

	Krippner			Wu-Xia		
Param.	Mean	Std	[0.05, 0.95]	Mean	Std	[0.05, 0.95]
π^*	1.83	0.090	[1.68, 1.98]	1.81	0.080	[1.69, 1.95]
σ	0.43	0.035	[0.38, 0.49]	0.37	0.042	[0.31, 0.44]
σ_{ν_1}	0.13	0.080	[0.057, 0.26]	0.14	0.093	[0.059, 0.26]
σ_{ν_2}	0.10	0.035	[0.057, 0.17]	0.18	0.084	[0.074, 0.34]
$\theta_{1,0}$	0.66	0.38	[0.063, 1.30]	0.73	0.37	[0.14, 1.36]
$\theta_{2,0}$	0.71	0.17	[0.43, 1.00]	0.74	0.19	[0.44, 1.06]

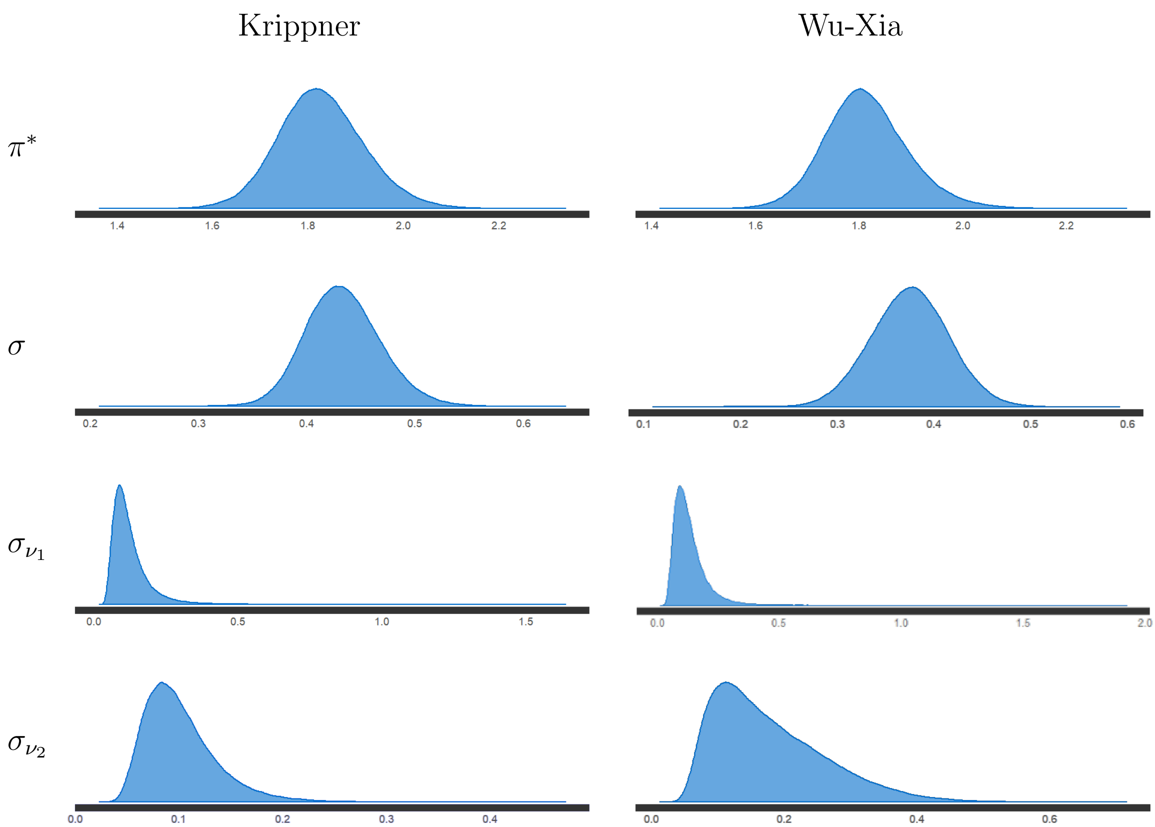
Notes: Posterior estimates of the time-invariant parameters and the initial values of the time-varying reaction coefficients obtained with the Krippner and the Wu-Xia shadow rate series, respectively, for the time period from 1999.2 to 2021.2. The corresponding 90 percent credible intervals are displayed in brackets. The priors are set according to the results in Table 1.

The sophisticated diagnostics unique to the HMC/NUTS algorithm did not signal any severe issues related to the sampling which would invalidate the results.[8]

[8] The Stan package comes along with a handy diagnostics interface, ShinyStan. An introduction to the diagnostics which is superior to those available to commonly applied MCMC-based algorithms, can be found in the references in the documentation available to Stan Development Team (2017).

In particular, there are no divergent sample draws at all. This indicates that the posterior likelihood surface does not exhibit any severe irregularities such as cliffs or areas with high local curvature which would prevent the sampler from exploring the posterior likelihood surface properly. In addition, the sampler did not reach the maximum tree-depth implying that the NUTS sampler could always traverse the posterior likelihood surface for an optimal distance along the Hamiltonian path. However, the energy level diagnostics, see Figure A.2 in the Appendix, suggests that the posterior exhibits heavy tails which may be challenging to sample from. Fig-

Figure 3: Posterior distribution estimates of time-invariant parameters



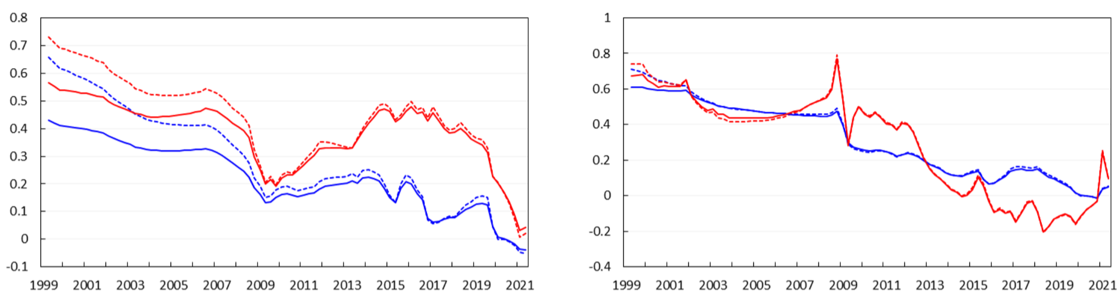
Notes: Posterior distribution estimates of the time-invariant parameters: inflation target, π^*, standard deviation of the error term in the policy rule, σ, and standard deviations of the random walk innovations governing the time-varying reaction coefficients, σ_{ν_1} and σ_{ν_2}. The priors are set according to the results in Table 1. Estimation period: 1999.2–2021.2.

ure 3 shows the marginal distributions of the posterior with respect to the inflation target, π^*, the standard deviation of the error term in the policy rule, σ, and the standard deviations of the innovations in the random walk process characterizing the reaction coefficients, σ_{ν_1} and σ_{ν_2}. The plots in the left column show the marginal distributions obtained with the Krippner series and the right column those obtained with the Wu-Xia series. While the marginal distributions of π^* and σ have a regular shape, the distributions of σ_{ν_1} appear to have long tails for both shadow rates series. Furthermore, the distribution of σ_{ν_2} obtained with the Wu-Xia shadow rate series is strongly skewed. The long tails in the distributions of σ_{ν_1} indicate that there is a positive probability for the response to the expected inflation gap to be far more volatile than initially expected. Long or heavy tails may postulate an issue for the sampler as the tail of the distribution requires a relatively large step size to be efficiently explored by the HMC algorithm while to deal with the higher local curvature of the posterior density surface in the region where the probability mass is concentrated a relatively small step size may be appropriate. A relatively large step size could result in unstable dynamics so that the sampler produces divergent draws which would invalidate the results while a relatively small step size may lead to a random walk behaviour. In the optimum, when the likelihood surface does not suffer from irregularities at all, the HMC algorithm produces nearly uncorrelated sample draws due to its gradient based approach and ability to cover large distances in the parameter space. However, in this case the step size was calibrated to be relatively small due to the high curvature in the central region of the posterior. The correlogramm of the sample draws for the conditional standard deviation of the response to the expected inflation gap, σ_{ν_1}, see Figure A.3 in the Appendix, exhibits a relatively high autocorrelation of the sample draws. This explains the large amount of sample draws required to achieve an effective sample size of at least 1,000. In this case, this does not constitute a major issue though, as the trajectories of the four different chains do not feature any obvious irregularities which would prevent the sampler from exploring the posterior likelihood surface properly. There is no indication that the chains would have stuck in a certain region and the scatter plots of σ_{ν_1}, σ_{ν_2} and σ do not show any obvious issues with identification either, such as a ridge, see Figure A.4 in the Appendix. They indicate, however, a slight bimodality which is more pronounced in case the Wu-Xia series is used for the estimation. However, this does not constitute an issue for the sampler either, as the energy barrier between the two regions is not as high that it would cause the sampler to stuck in one of the modes. In general, long tails do not constitute an issue, as the sampling algorithm is feasible and given the sufficiently high effective sample size, there are no irregularities which would invalidate the results.[9]

5. Robustness checks and extensions

No model training. A straightforward question to be asked is how results change depending on the priors assumed. I estimated the model on the entire data sample from 1999.2 to 2021.2 directly with the priors from Section 2 without conducting any training or subsequent adjustments in the priors. Figure 4 shows the results in comparison to the baseline estimates from Section 4 with priors based on the results in Table 1. The main difference in both estimates occurs in the estimate for the

Figure 4: Time-varying response from 1999.2 to 2021.2 without model training



(a) Expected Inflation Gap (b) Expected Growth Gap

Notes: Plot (a) shows the estimates for the time-varying response to the expected inflation gap, $\theta_{1,t}$, and plot (b) to the expected growth gap, $\theta_{2,t}$. The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate series, respectively. The solid lines correspond to the results obtained with the priors from Section 2 and the dashed lines display the baseline estimates from Section 4 with priors according to the results in Table 1.

response to the expected inflation gap. For both shadow rate series, the response to the expected inflation gap at the initial point, $\theta_{1,0}$, is markedly lower than in the baseline estimation. The lower estimate for $\theta_{1,0}$ alters the slope of the trajectory of

[9]For the detailed Stan output containing the effective sample sizes see Table B.2 and B.3 in the Appendix.

$\theta_{1,t}$, yet the difference to the baseline estimate gradually vanishes until 2008.4. The lower estimates until 2008.4 are most likely driven by the significantly lower prior value set for $\theta_{1,0}$, in particular 0.50 instead of 0.90. Consequently, the estimate for the response at the beginning depends significantly on the prior belief about the commitment to keep inflation close to the target at the inception of monetary union. The standard deviation of the estimation is larger though as shown in Table 3. The

Table 3: Priors and posterior estimates without model training – 1999.2-2021.2

	Prior			Krippner			Wu-Xia		
Par.	Dens.	Mean	Std	Mean	Std	[0.05, 0.95]	Mean	Std	[0.05, 0.95]
π^*	Norm	1.9	0.15	1.83	0.12	[1.64, 2.04]	1.81	0.11	[1.66, 2.00]
σ	InvG	0.2	0.20	0.45	0.038	[0.40, 0.52]	0.39	0.045	[0.32, 0.47]
σ_{ν_1}	InvG	0.2	0.20	0.13	0.067	[0.051, 0.23]	0.13	0.078	[0.054, 0.25]
σ_{ν_2}	InvG	0.2	0.20	0.10	0.038	[0.05, 0.17]	0.18	0.092	[0.067, 0.36]
$\theta_{1,0}$	Norm	0.5	2.00	0.43	0.49	[-0.32, 1.29]	0.56	0.51	[-0.22, 1.44]
$\theta_{2,0}$	Norm	0.5	2.00	0.61	0.29	[0.15, 1.09]	0.67	0.41	[0.042, 1.36]

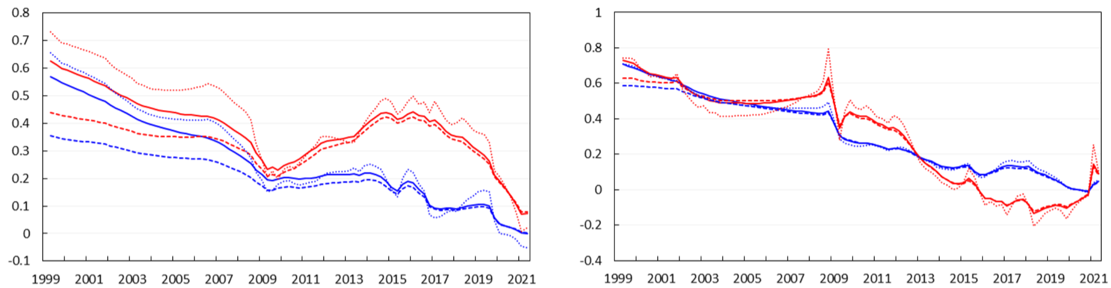
Notes: The posterior means and standard deviations of the time-invariant parameters as well as the initial values of the time-varying parameters were estimated on the complete data sample from 1999.2 to 2021.2 without any estimation of the model on a training sample. The corresponding 90 percent credible intervals are displayed in brackets.

estimate for $\theta_{1,0}$ obtained with the baseline setup in Section 4 lies also well within the 66 percent credible interval of this estimate. The difference between the estimates for $\theta_{1,t}$ is negligible from 2008.4 onwards. This implies that for the period after the financial crisis the data pins down the time-varying parameters fairly well. This could be attributed to the larger variation in the shadow rate series than in the MRO. The estimation with respect to the response to the expected growth gap is very small throughout the entire period for $\theta_{2,t}$. However, there is some slight difference at the very beginning of the sample if compared with the baseline estimation results which vanishes very rapidly. Similarly to $\theta_{1,0}$, this is also very likely to be attributable to the smaller prior assumed for $\theta_{2,0}$, 0.50 against 0.76 in the baseline setup. As regards the time-invariant parameters, see Table 3, the estimates for the conditional standard deviations of the random walk innovations, σ_{ν_1} and σ_{ν_2}, and for the inflation target, π^*, are almost similar to those obtained with the baseline setup from Table 2. The estimates for the standard deviation of the error term in the policy rule, σ, are slightly larger than before, 0.45 as opposed to 0.43 and 0.39 to 0.37. Therefore, the variation of the estimated time-varying response to the expected inflation and growth gap remains similar. As regards the diagnostics, there were no divergent transitions which would invalidate the results, similarly to the other estimations. However, a very large amount of sample draws is still needed to achieve an acceptably high effective sample size because of the long tails in the marginal distributions of σ_{ν_1} and σ_{ν_2}.

Less varying response. To assess how results change when less variation is permitted in the reaction coefficients I set a tighter prior distribution along with a lower expected value for the standard deviations of the innovations, σ_{ν_1} and σ_{ν_2}. I assume that each of the latter two parameters is again inverse gamma distributed, yet with an expected value of 0.1 and a standard deviation of 0.05 instead of assuming 0.2 for both the expected value and the standard deviation as in Section 2. The rest of the priors corresponds to those in Section 2 and in the previous subsection. I estimated the modified model first by following the baseline procedure, that is, I estimated it on the training sample from 1999.2 to 2001.4 and then set the obtained results as new priors to finally rerun the estimation on the complete data sample from 1999.2 to 2021.2. Afterwards I also carried out the estimation using the complete data sample without training the model on any data sample.

Figure 5 shows that the tighter prior with the lower expected value renders the reaction coefficients smoother, as expected. A further consequence of the reduced variation is that the response to the expected inflation gap is initially lower than in the baseline estimation. Yet, the difference between the curves vanishes. In case the model is estimated without training, the initial difference is more pronounced owing to the markedly smaller priors for the starting values, $\theta_{1,0}$ and $\theta_{2,0}$. The difference in the response to the expected growth gap compared to the baseline results is much smaller than in case of the expected inflation gap. The response to the expected growth gap is essentially the same with the Krippner series and is slightly larger with the Wu-Xia series in the period between 2002 and 2008, whereas at the peak in

Figure 5: Time-varying response with tight priors from 1999.2 to 2021.2



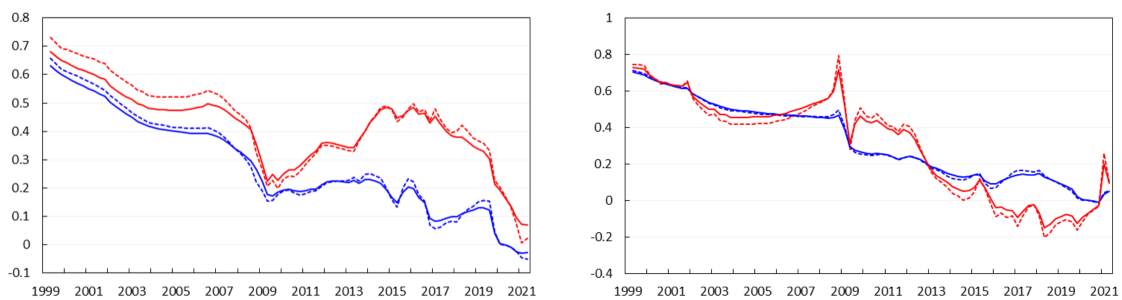
(a) Expected Inflation Gap (b) Expected Growth Gap

Notes: The blue and the red lines refer to the results obtained with the Krippner and the Wu-Xia shadow rate series, respectively. The solid lines show the estimates from the tight prior setup, yet obtained by training the model first on the data sample from 1999.2 to 2001.4, while the dashed lines refer to the estimates on the complete data sample from 1999.2 to 2021.2 without any model training. The dotted lines display the baseline estimates from Section 4 for comparison purposes.

2008.4 it is smaller. Afterwards the results do not differ much from those obtained with the baseline setup. The posterior mean of the inflation target ranges between 1.83 and 1.84 which is slightly higher for the Wu-Xia series while it remains essentially the same for the Krippner series, see Table B.4 and B.5 in the Appendix. The volatility of the deviation from the policy rule is very slightly higher if compared with the results from the baseline setup. It is estimated at 0.44 and 0.46 for the Krippner series and at 0.40 and 0.42 for the Wu-Xia series, compared to 0.43 and 0.37 from Table 2. This suggests that the volatility in the data translates into higher deviations from the policy rule. In general, one can conclude that the main difference lies in the estimates for the response to the expected inflation gap in the initial period. Again, no issues were signaled by the diagnostics. The effective sample sizes of σ_{ν_1} and σ_{ν_2} improved slightly owing to the shorter tail in the marginal distribution, however compared with the rest of the parameters it is still very low.

Correlated response. A further canonical extension of the model is to allow for a cross correlation between the error terms of the time-varying reaction coefficients. Hence, I release the assumption that Q is diagonal. I will maintain though the assumption that the initial states, θ_0, are independently distributed and also independent from the rest of the priors. Therefore, I can assume that the prior distribution of Q is Inverse-Wishart with $\sim \mathcal{W}^{-1}(\text{diag}(0.18^2, 0.16^2), 4)$. Thereby, I leave Q relatively unrestricted and let the data decide about the cross-covariances. For the rest of the priors the results from Table 1 are used. Figure 6 shows that allowing for cross-correlation changes the evolution of the response to both the expected inflation gap and growth gap over time only very slightly, compared to the baseline results. Yet, the estimates suggest a slight positive cross correlation between the innovations in the reaction coefficients with 0.17 and 0.08 for the Krippner and the Wu-Xia series, respectively. Using the priors from Section 2 for the rest of the parameters results in an even lower cross correlation of 0.10 and 0.05 for the two datasets, respectively, while results remain almost similar to the case where no model training is carried out. The corresponding results are shown in Figure A.6 in the Appendix.

Figure 6: Correlated time-varying response from 1999.2 to 2021.2



(a) Expected Inflation Gap (b) Expected Growth Gap

Notes: The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate series, respectively. The solid lines correspond to the results obtained by allowing for correlation between the innovations in the random walk processes governing the reaction coefficients. The dashed lines display the baseline estimates from Section 4 for comparison purposes.

Stochastic volatility. To account for periods of both higher and lower variation in the deviation from the policy rule I extend the model by assuming stochastic volatility for the error term in the policy rule, so I postulate the following structure for the disturbance in the observation equation:

$$\epsilon_t := \gamma_t e^{\frac{s_t}{2}}, \ \gamma_t \sim N(0, 1). \tag{19}$$

The volatility process, s_t can be described as follows:

$$s_t = \mu_s + \rho_s(s_{t-1} - \mu_s) + \psi_t \tau_s \tag{20}$$

$$\psi_t \sim N(0,1), \quad s_0 \sim N\left(\mu_s, \frac{\tau_s^2}{1-\rho_s^2}\right) \tag{21}$$

where ψ_t is the shock to the volatility and τ_s the corresponding scaling parameter. Combining the equations, the conditional distribution of s_t is given by

$$(s_t|s_{t-1}, \mu_s, \rho_s, \tau_s) \sim N\left(\mu_s + \rho_s(s_{t-1} - \mu_s), \tau_s^2\right). \tag{22}$$

The conditional distribution of ϵ_t equals to:

$$(\epsilon_t|s^T, \mu_s, \rho_s, \tau_s) = (\epsilon_t|s_t, \mu_s, \rho_s, \tau_s) \sim N\left(0, \upsilon_t^2\right) \tag{23}$$

with $\upsilon_t := e^{\frac{s_t}{2}}$. Since I assume that there is no cross correlation between the error terms of the state space, the likelihood translates to

$$p(Y^T|\tilde{\theta}^T, \sigma_{\nu_1}, \sigma_{\nu_2}, \mu_s, \rho_s, \tau_s, s^T) = \prod_{t=0}^{T} \frac{1}{(2\pi\upsilon_t^2)^{1/2}} \left[\exp\left\{-\frac{(y_t - f(X_t, \tilde{\theta}_t))^2}{2\upsilon_t^2}\right\}\right]. \tag{24}$$

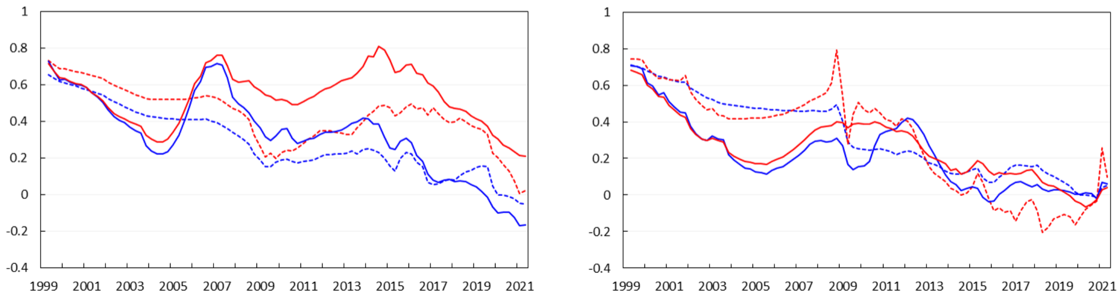
In addition, I assume that all parameters determining s_t are independently distributed from the rest of the parameters. As a consequence, the joint prior distribution can be written as

$$\begin{aligned}
&p(\tilde{\theta}^T, \sigma_{\nu_1}, \sigma_{\nu_2}, \mu_s, \rho_s, \tau_s, s^T) = p(\theta^T|\pi^*, \sigma_{\nu_1}, \sigma_{\nu_2}, \mu_s, \rho_s, \tau_s, s^T)p(\pi^*, \sigma_{\nu_1}, \sigma_{\nu_2}, \mu_s, \rho_s, \tau_s, s^T) = \\
&p(\pi^*)p(\sigma_{\nu_1})p(\sigma_{\nu_2})p(\mu_s, \rho_s, \tau_s)p(s^T|\mu_s, \rho_s, \tau_s)p(\theta_0) \prod_{t=0}^{T-1} p(\theta_{t+1}|\theta_t, \sigma_{\nu_1}, \sigma_{\nu_2}) = \\
&p(\pi^*)p(\sigma_{\nu_1})p(\sigma_{\nu_2})p(\mu_s)p(\rho_s)p(\tau_s)p(s_0)p(\theta_0) \prod_{t=0}^{T-1} p(s_{t+1}|s_t, \mu_s, \rho_s, \tau_s) \prod_{t=0}^{T-1} p(\theta_{t+1}|\theta_t, \sigma_{\nu_1}, \sigma_{\nu_2}).
\end{aligned} \tag{25}$$

To estimate the model I assume that ρ_s is uniformly distributed in the interval $(-1, 1)$ and both μ_s and τ_s are Cauchy distributed with $C(0, 5)$[10]. Figure 7 shows the results in comparison to those from the baseline estimation. The first main difference is that

[10]The probability density function of the Cauchy distribution is given by $p(x; a, b) = \frac{1}{\pi b\left[1+\left(\frac{x-a}{b}\right)^2\right]}$.

Figure 7: Time-varying response with stochastic volatility from 1999.2 to 2021.2



(a) Expected Inflation Gap

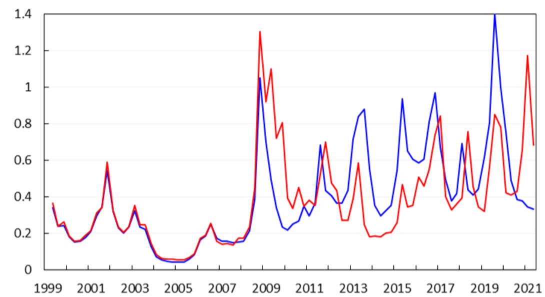
(b) Expected Growth Gap

Notes: The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate rate series, respectively. The solid lines show the estimates of the time-varying reaction coefficients assuming stochastic volatility in the deviations from the policy rule. The dashed lines display the baseline estimates from Section 4 for comparison purposes.

until the beginning of the financial crisis the response to the expected inflation gap is far more volatile than without modelling stochastic volatility. It decreases until end-2004 and increases then sharply until mid-2007. Although inflation hovered around the target until the financial crisis, the response was considerably weaker for a relatively short period than initially suggested by the baseline model. This finding is also supported by the response to the expected growth gap which decreases as well until end-2004 and increases sharply afterwards until the outbreak of the financial crisis. Given that the policy rate remained constant between 2003.3 and 2005.4 these results are hardly surprising. After the beginning of the financial crisis the response to the expected inflation gap depends strongly on the data series used. The estimation based on the Krippner series exhibits the same pattern as in the baseline estimation. It decreases continuously until the end of the sample period where it reaches again negative values. In contrast, the results obtained with the Wu-Xia series indicate that after a relatively moderate decline the response strengthens and reaches a new peak. However, afterwards it decreases continuously and ends up at a relatively low level, yet not zero. The pattern of the response to the expected growth gap following the start of the financial crisis is comparable with that from the baseline estimation irrespective of the data series used. For the Krippner series initially it decreases but rebounds then relatively fast and peaks in the second half of

2012, yet drops to zero in the aftermath. For the Wu-Xia series it declines smoothly and drops to roughly zero towards the end of the sample.

Figure 8: Posterior estimates of time-varying volatility from 1999.2 to 2021.2



Notes: The blue and the red lines refer to the evolution of the of the volatility in the deviations from the policy rule obtained with the Krippner and with the Wu-Xia shadow rate rate series, respectively.

The estimated evolution of the volatility, see Figure 8, suggests that until the beginning of the financial crisis the deviation from the estimated response tended to be smaller and less volatile. Allowing for a varying volatility in the deviations from the policy rule renders the reaction coefficient smaller which provides a better fit. In contrast, with a constant volatility parameter the larger deviations from the policy rule following the financial crisis increase the volatility of the deviations uniformly also for the period before the financial crisis. This results in a more forceful estimated response along with a larger deviation from the policy rule whereas in reality the response might have been less forceful and the deviations from it small. Therefore, it is essential to account for changes in the volatility over time.

In the aftermath of the financial crisis the magnitude of the deviations increased remarkably as with non-standard measures there is obviously less room for a fine tuning of the policy rate. These results imply that the central bank may have been more aggressive for quiet some time after the financial crisis than initially estimated without assuming changes in the volatility in the deviations from the policy rule over time. Since the central bank does not have the ability to set policy rates directly, deviations from the assumed response are larger. Therefore, for the agents in the economy it will be more difficult to distinguish between an actual change in the systematic response and a larger deviation from the policy rule. This may render policy less efficient, as agents may build expectations relying on inaccurate values with respect to the strength of the response. A further notable result is that although technically s_t was restricted to be stationary, there might be little need to allow for permanent changes in the magnitude of the deviations from the policy rule as ρ_s is estimated at 0.75 and 0.66 for the Krippner and the Wu-Xia series, respectively. This implies that deviations from the intended policy response occur only on a temporary basis, typically when the economy is hit by large shocks, for example at the beginning of the financial crisis or at the outbreak of the recent pandemic.

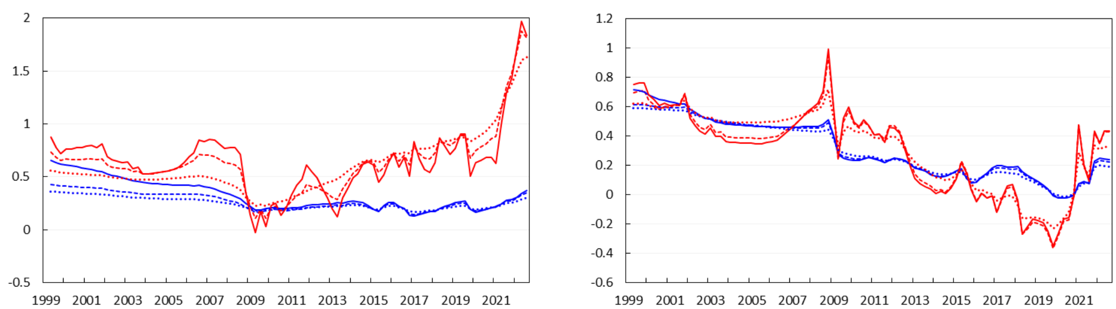
The diagnostics did not signal any serious issues with sampling. However, the estimation of the model with the Krippner series resulted in one single diverging sample draw within overall 200,000 sample draws. This means that in a single case when the new proposal draw was calculated by applying the numerical solution algorithm to the Hamiltonian equation it drifted off the Hamiltonian path. Usually this occurs when the local curvature of the joint posterior is very large for the calibrated step size. However, in case of this more sophisticated model a single divergent sample draw can be ignored as the relative amount of divergent sample draws is marginal if compared with the overall number sample draws. Moreover, the one divergent sample draw does not exhibit any pattern that would indicate that the algorithm is unstable in a certain region and therefore the sampler cannot access certain regions of the joint posterior. To remedy this issue it is recommended to reduce the step size of the HMC algorithm. However, to achieve this result the step size had to be reduced further manually by an immense magnitude compared to the automatically calibrated value by the software package used. This indicates, that the expansion of the model with stochastic volatility results in a way more complex joint posterior surface which is more complicated to be sampled from and increases the runtime immensely.[11] Using common non-gradient based sampling methods, as the random

[11] Without modelling stochastic volatility the runtime is typically less than 5 minutes on a commonly available desktop computer equipped with an AMD 3950x CPU. The expansion of the model with stochastic volatility increased the runtime to 2-3 days in order to obtain 200,000 sample draws with only one divergent draw. With the Wu-Xia series there were no divergences among the 200,000 sample draws. However, the step size had to be reduced approximately by the same magnitude and resulted in a very similar runtime as for the estimation with the Krippner series.

walk Metropolis Hastings algorithm, it is questionable whether sampling would be tractable at all. Furthermore, without sophisticated diagnostic features unique to the HMC algorithm it could easily occur that one would not even realize that the sampler could not access certain parts of the posterior and end up with biased results.

New inflation target. In course of its strategy review in 2021 the ECB has announced its new symmetric inflation target of 2 percent. Therefore, I assume that from 2021.3 onwards π^* equals $2+d$ percent, where d is assumed to be normally distributed with $d \sim N(0, 0.1^2)$. Given the recent inflation developments and to achieve a better identification I truncate d at zero and exclude thereby negative values. Redefining $\tilde{\theta}^T$ by adding d, the arguments from Section 2 hold, so the joint posterior can be expanded by the prior distribution of d. I estimated the model first without assuming stochastic volatility. Figure 9 shows that the response to the expected

Figure 9: Time-varying response from 1999.2 to 2022.3



(a) Expected Inflation Gap (b) Expected Growth Gap

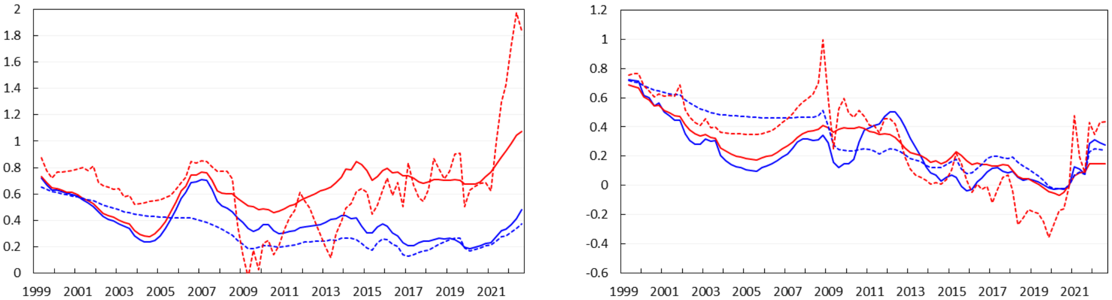
Notes: Plot (a) shows the estimates for the time-varying response to the expected inflation gap, $\theta_{1,t}$, and plot (b) to the expected growth gap, $\theta_{2,t}$. The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate rate series, respectively. The solid lines refer to the estimates using priors based on Table 1. The dashed lines refer to the estimates using priors from Section 2. The dashed lines refer to the estimates using priors from Section 2 with the tight prior setup for the standard deviations of the innovation in the time-varying parameters.

inflation gap is considerably influenced by the recent response to its sharp rise. For the Krippner series, the estimates in the period until the start of the financial crisis depend again on which initial priors are assumed as in the baseline estimation until 2021.2. However, if compared with the results from the baseline setup, after 2017 the reaction function does not decrease anymore, but increases again except a short interruption between 2019.4 and 2020.1. At the end of the sample period the response to the expected inflation gap has reached its highest value since 2007.3. The fact that towards the end of the sample period the estimates are influenced substantially by the most recent data is inherent in the random walk structure of the reaction coefficients. In periods when the time-varying parameters are less well identified or the relationship with the dependent variable might be weak, the values are influenced considerably by the parameter values in those periods with a better identification and a stronger relationship with the dependent variable. For the Wu-Xia series the first main difference is that the model prefers a more volatile response to the expected inflation gap over time. Moreover, the strong response at the end of the sample period influences the values significantly from approximately 2012 onwards. Following the decrease before the outbreak of the pandemic the response to the expected inflation gap increases sharply and reaches even more than two times higher values than before the start of the financial crisis. Setting even tighter priors for the conditional standard deviation of the reaction coefficients does not alter the main pattern, yet renders the evolution smoother. As regards the response with respect to the expected growth gap the evolution is very similar if compared with the baseline results using data only until 2021.2. Afterwards the response increases slightly, yet it still remains in relatively low ranges.

In light of the recent rise in both realized and expected inflation a natural question to be asked is the whether recently the ECB has become more tolerant against inflation and to what extend the true inflation target has changed. For the period until 2021.2 the inflation target with the Krippner series is estimated at 1.82 being essentially similar to the results from the baseline setup. Using the Wu-Xia series the target is estimated to be 1.74 percent which is somewhat lower than the baseline estimate. As regards the short period following the strategy review, the parameter d is estimated at 0.08 and 0.06 for the Krippner and the Wu-Xia series, respectively. This suggests that the inflation target is estimated at very slightly above the symmetric 2 percent target.

The model was also reestimated with the stochastic volatility setup from above.

Figure 10: Time-varying response with stochastic volatility from 1999.2 to 2022.3



(a) Expected Inflation Gap

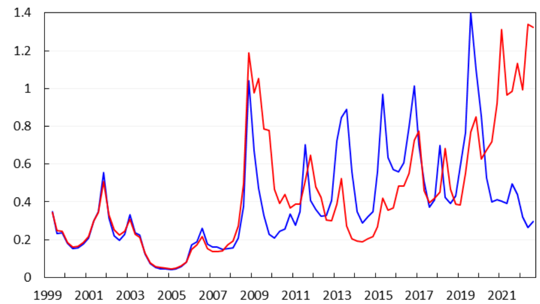
(b) Expected Growth Gap

Notes: Posterior estimates of the time-varying reaction coefficients assuming stochastic volatility in the deviations from the policy rule. The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate rate series, respectively.

Figure 10 suggests that with the Krippner series the response to the expected inflation gap exhibits a continuously falling tendency from the beginning of 2014 and reaches the trough only in 2020.1, while without stochastic volatility it appears to have been increasing gradually since 2017. In addition, from 2020.2 the increase is steeper and ends up at 0.48 at the end of the sample period being higher than the estimated value of 0.38 without modelling stochastic volatility. For the Wu-Xia series, the stochastic volatility setup renders the evolution of the response to the inflation gap relatively smooth. Moreover, the increase from 2020.2 is far less steep than without assuming stochastic volatility. It is also considerably lower even if compared with the case from Figure 9 where tight priors are used for the standard deviation of the innovations to prevent the time-varying reaction process from sudden fluctuations. Nevertheless, the response reaches its highest value ever at the end of the sample period. As regards the response to the expected growth gap, using the Krippner series it is slightly more volatile which results in turn in a slightly higher response towards the end of the sample period than without modelling stochastic volatility. For the Wu-Xia series the response is far less volatile than without stochastic volatility and notably lower at the end of the sample period.

Finally, the estimation results for the volatility suggest that for the Krippner series the deviations from the policy rule have fallen back to levels prevailing before the financial crisis following the immense pandemic shock. The results with respect

Figure 11: Posterior estimates of time-varying volatility from 1999.2 to 2022.3



Notes: The blue and the red lines refer to the evolution of the of the volatility in the deviations from the policy rule obtained with the Krippner and with the Wu-Xia shadow rate rate series, respectively.

to the Wu-Xia series suggest though that the volatility has kept rising and reached its highest levels ever. As regards the persistence of the changes in the magnitude of the volatility in the deviations from the policy rule, for the Krippner series ρ_s is slightly lower with 0.73 than the estimate without considering the post strategic review period. In contrast, for the Wu-Xia series it increases from 0.66 to 0.80. The latter result suggests, that deviations from the policy rule may have become recently longer lasting. Thus, monetary policy may remain more aggressive than the policy rule would suggest. In the future this could translate in a higher estimated systematic response.

6. Conclusion

In order to assess the changes in the monetary policy stance of the euro area since its inception I estimated a monetary policy function, which has been shown to adequately describe the ECB's policy response (see e.g. OW, BW and HS). To that end, I developed a Bayesian time-varying parameter framework and sampled from the joint posterior distribution of the parameters by using the Hamiltonian Monte Carlo algorithm. I also improved on existing studies by relying on two different shadow interest-rate series instead of key policy rates. That permitted to estimate the impact of non-conventional measures on the ECB's reaction function directly during periods when monetary policy may have been constrained by the effective lower bound. In addition, this flexible framework allowed for a more sophisticated modelling of the stochastic volatility associated with deviations from the policy rule.

I found that the ECB's response to the expected inflation gap and the expected growth gap has varied considerably over time. While the response to the expected growth gap has weakened substantially in the aftermath of the financial crisis, the estimation results for the response to the expected inflation gap are somewhat mixed and depend on the shadow rate series used for estimation purposes. In particular, the majority of the results suggests that the response weakened since the onset of the financial crisis and when quantitative measures were in place. There are also indications, however, that this weakening in the response to expected inflation was less pronounced.

If the period following the ECB's strategy review is also included into the data sample used for the estimation, the results imply that the ECB's response has recently become more forceful. However, while the results obtained with Krippner's shadow rate series suggest that the response to the expected inflation gap remains well below its historical peak, the results obtained by using the alternative shadow rate series provided by Wu and Xia show that it recently achieved its highest level ever. The estimation results obtained with the extended data sample including the post-strategy review period also indicate that the response to the expected inflation gap weakened much less during the years prior to the pandemic, especially if the model is estimated with the Wu-Xia shadow rate series. This feature is inherent in the model because periods of weak responses, or which may be less well identified by the data, are strongly affected by subsequent peaks in the response. This may make the estimated policy response more forceful during periods when the response was indeed weak. I also found that the inflation target rose to slightly above 2 percent as a result of the ECB's recent strategy review. Consequently, the results do not suggest much greater tolerance of a large deviation from the new target.

Factoring stochastic volatility into deviations from the policy rule over time is essential as it strongly influences the estimation results. Firstly, results suggest that the policy response in the run-up to the financial crisis was more volatile than that suggested by the baseline estimation setup. It declined significantly from its initially high levels until 2005 but then rebounded sharply. Secondly, the response to the expected inflation gap after the financial crisis was more forceful than it would have been without modelling stochastic volatility. Thirdly, the response to the expected inflation gap declined more sharply after quantitative easing was introduced in 2015.1. And, lastly, the recent increase in the response to the sharp rise in the expected inflation gap is much smaller, especially when the Wu-Xia shadow rate series is used for estimation, while with the Krippner series it is only slightly higher.

Given the importance of both the time variability of the response parameters and the stochastic volatility in the deviations from the policy rule, it is essential to take account of both when investigating possible changes in the stance of monetary policy. The HMC algorithm is a suitable approach for dealing with extremely complex and very high-dimensional models. Consequently, extensions of the present model in terms of its statistical and structural properties are easily possible to improve accuracy and structural interpretation. I leave these topics for future research.

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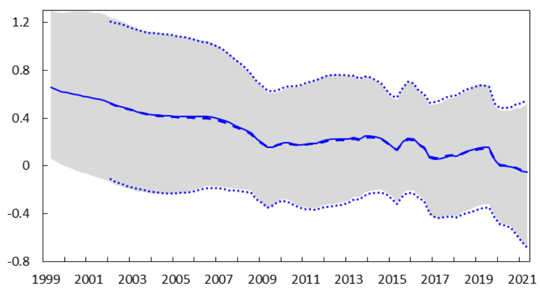
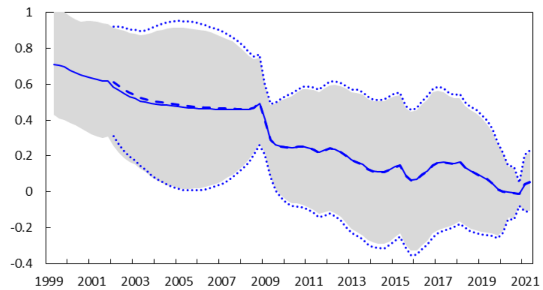
Appendix A. Figures

Figure A.1: Posterior estimates of time-varying reaction coefficients

Krippner

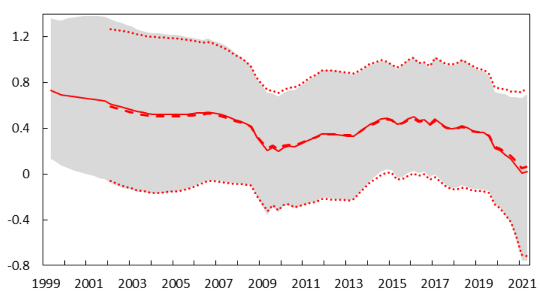


(a) Expected Inflation Gap



(b) Expected Growth Gap

Wu-Xia



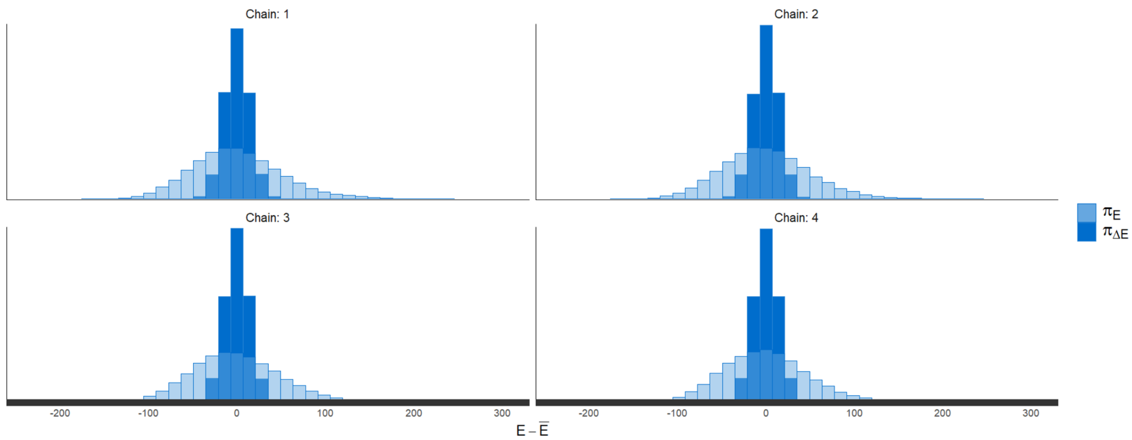
(c) Expected Inflation Gap

(d) Expected Growth Gap

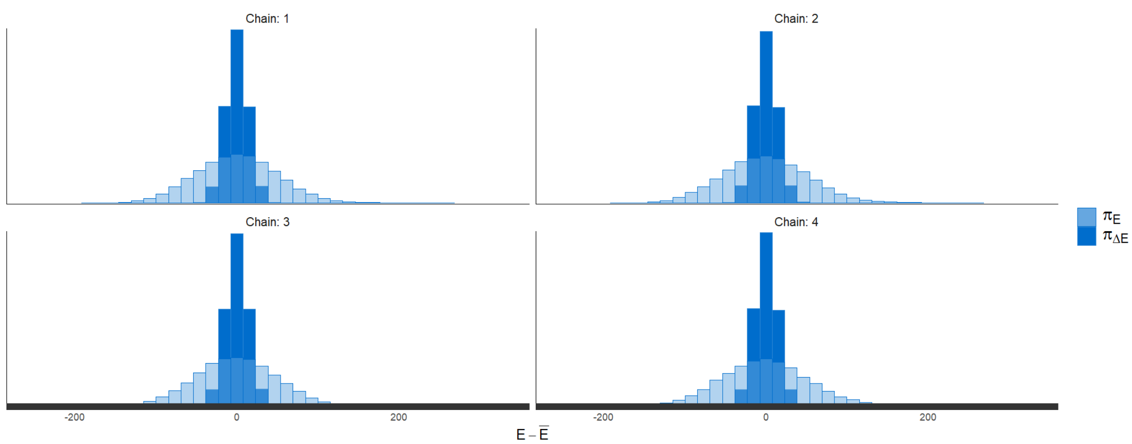
Notes: Plot (a) and (b): estimation results obtained with the Krippner shadow rate series. Plot (c) and (d): estimation results obtained with the Wu-Xia shadow rate series. The solid lines refer to the estimation on the complete data sample from 1999.2 to 2021.2 and dashed lines to the sample from 2002.1 to 2021.2. The grey shaded area and the dotted lines show the 90 percent credible intervals with respect to the estimation from 1999.2 to 2021.2 and from 2002.1 to 2021.2, respectively.

Figure A.2: HMC energy diagnostics

(a) Krippner

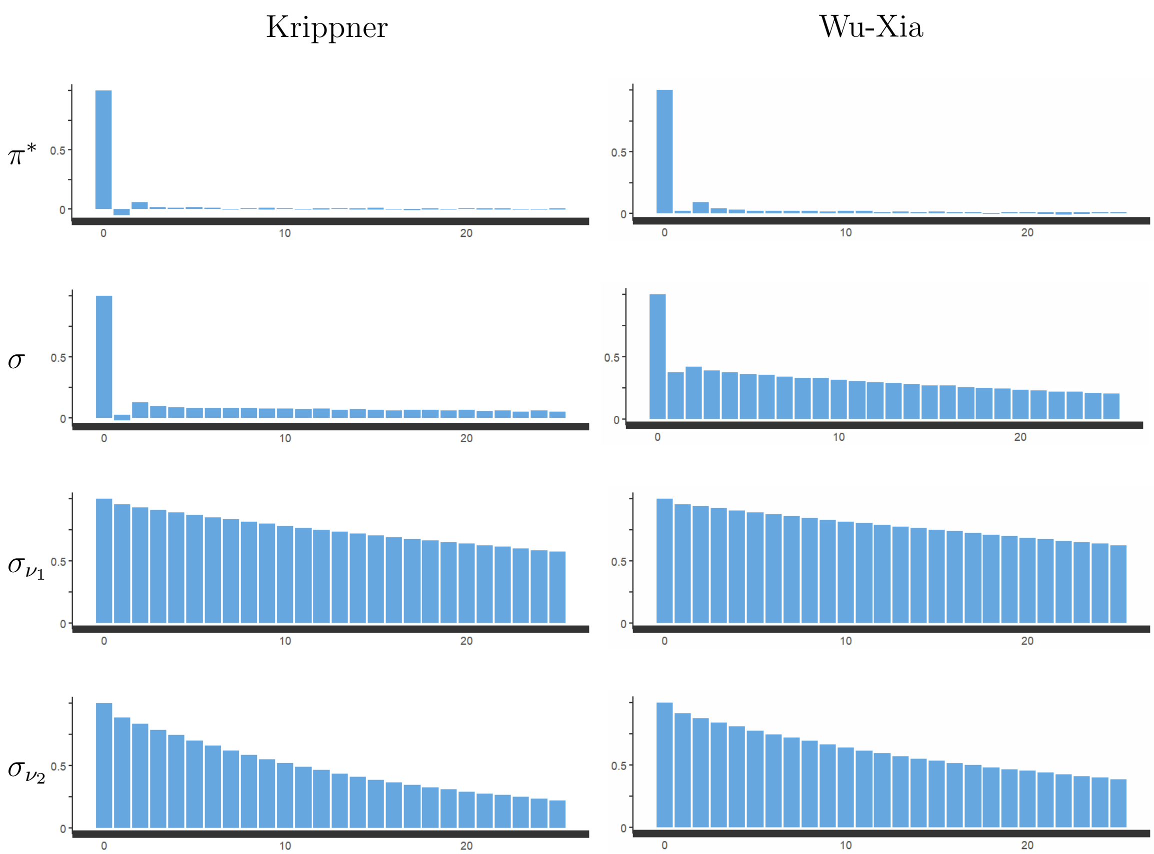


(b) Wu-Xia



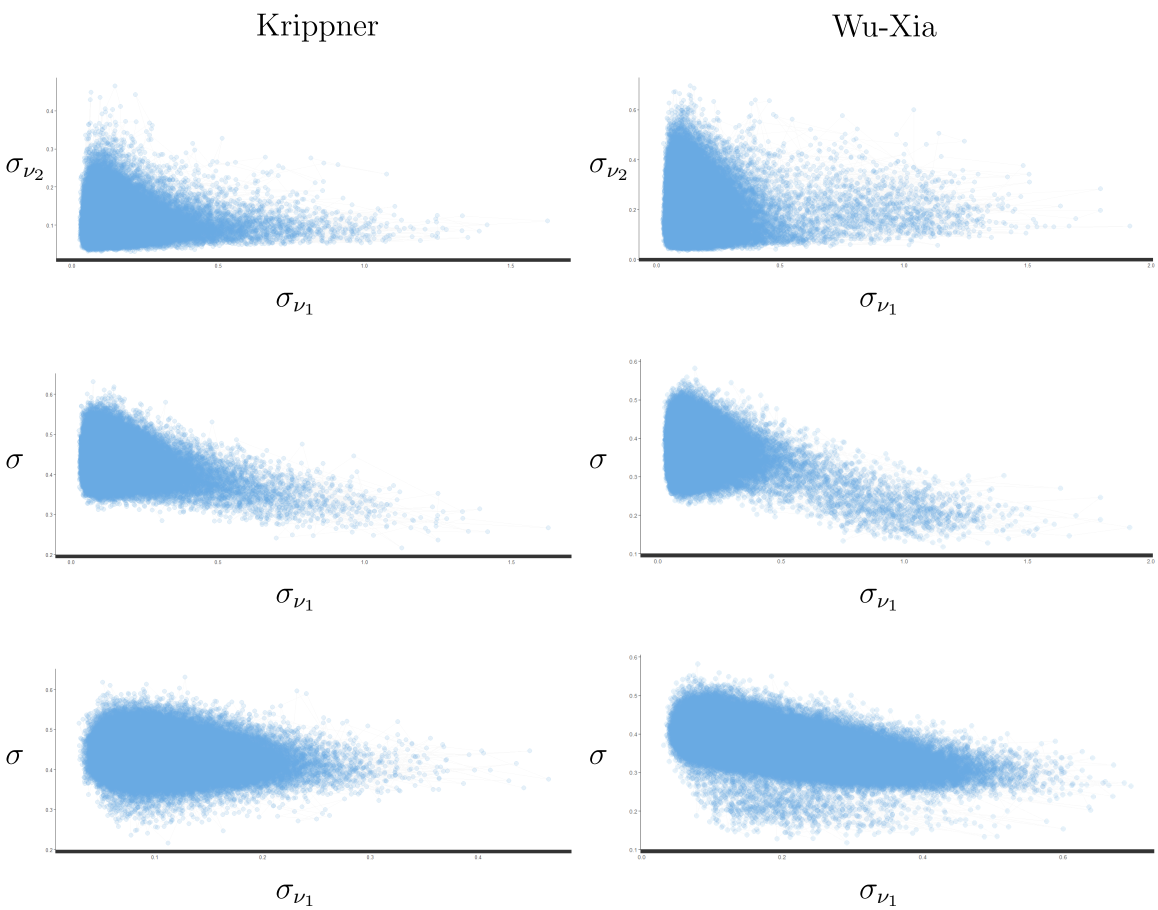
Notes: The figure shows the overlaid histograms of the (centered) marginal energy distribution and the first-differenced distribution which is referred to as the energy diagnostics. Plot (a) displays the diagnostics of each chain obtained with the Krippner shadow rate series while plot (b) shows the diagnostics of each chain obtained with the Wu-Xia shadow rate series. The estimation was carried out with data covering the period from 1999.2 to 2021.2.

Figure A.3: Correlogramms of time-invariant parameter sample draws



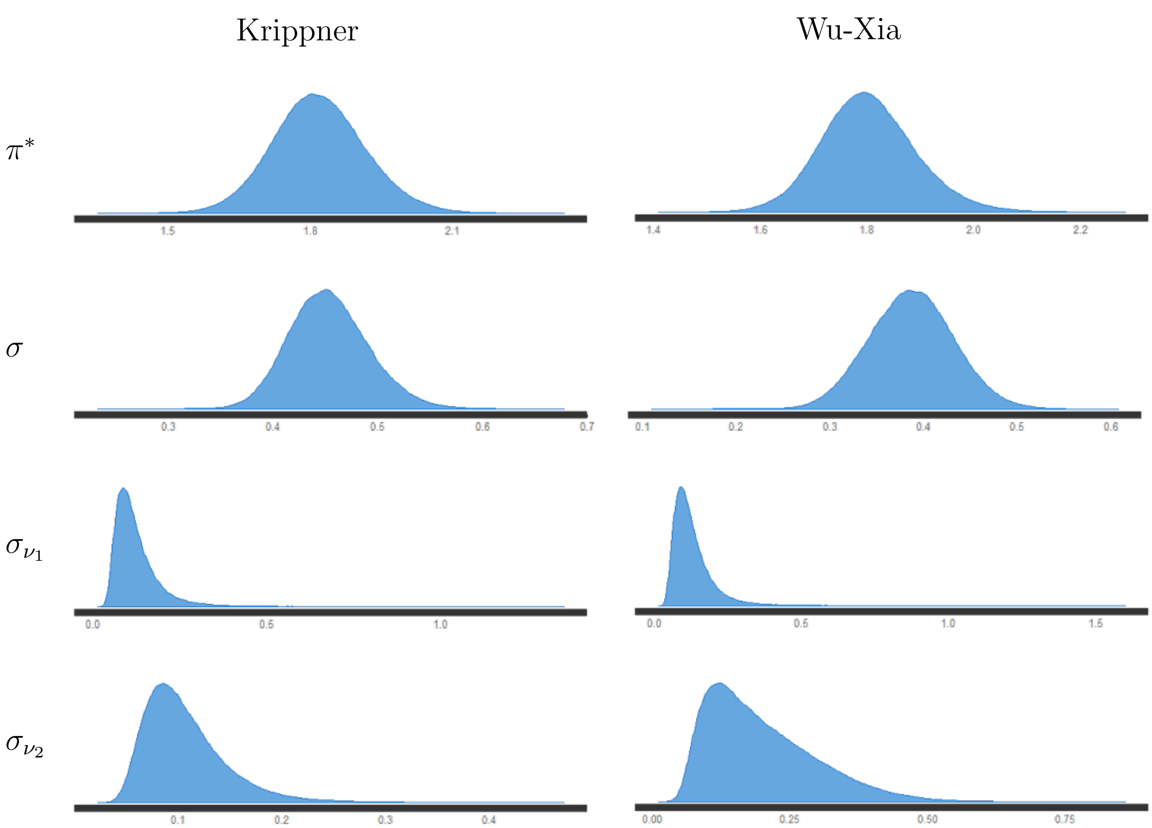
Notes: The plot shows correlogramms of the sample draws for the time-invariant parameters. The left and the right column contain the results obtained with the Krippner shadow rate series and with the Wu-Xia shadow rate series, respectively. Estimation period: 1999.2-2021.2.

Figure A.4: Scatter plots of sample draws of standard deviations



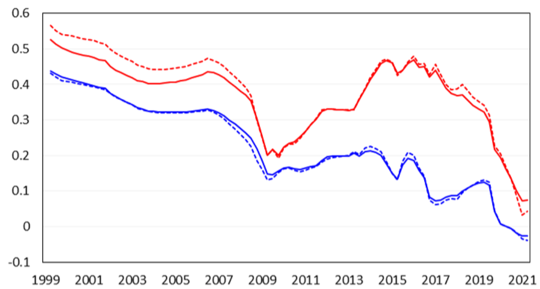
Notes: Sample draws of the standard deviations of the random walk innovations, σ_{ν_1} and σ_{ν_2}, and the standard deviation of the error term in the policy rule, σ.

Figure A.5: Posterior distributions of time-invariant parameters – 2002.1-2021.2

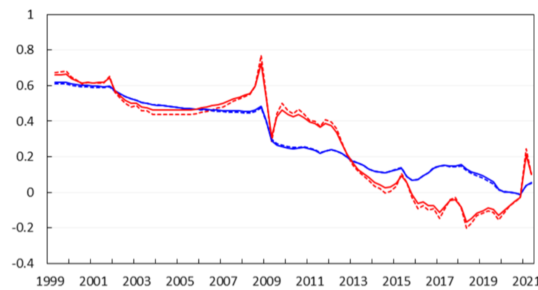


Notes: Posterior distributions estimates of the time-invariant parameters: inflation target, π^*, standard deviation of the error term in the policy rule policy rule, σ, standard deviations of the innovations in the processes governing the time-varying reaction coefficients, σ_{ν_1} and σ_{ν_2}. Priors are set according to the results from Table 1. Estimation period: 2002.1–2021.2.

Figure A.6: Correlated time-varying response without model training from 1999.2 to 2021.2



(a) Expected Inflation Gap



(b) Expected Growth Gap

Notes: The blue and the red lines refer to the results obtained with the Krippner and with the Wu-Xia shadow rate series, respectively. The solid lines correspond to the results obtained by allowing for correlation between the innovations of the random walk processes governing the reaction coefficients without any model training. The dashed lines display the baseline setup without any model training from Section 5 for comparison purposes.

Appendix B. Tables

Table B.1: Posterior estimates for the time period from 2002.1 to 2021.2

Param.	Density	Krippner Mean	Krippner Std	Krippner [0.05, 0.95]	Wu-Xia Mean	Wu-Xia Std	Wu-Xia [0.05, 0.95]
π^*	Normal	1.82	0.098	[1.66,1.98]	1.80	0.088	[1.67,1.95]
σ	InvGamma	0.45	0.038	[0.39,0.52]	0.39	0.046	[0.31,0.46]
σ_{ν_1}	InvGamma	0.12	0.070	[0.057,0.25]	0.13	0.082	[0.058,0.26]
σ_{ν_2}	InvGamma	0.11	0.039	[0.059,0.18]	0.20	0.097	[0.078,0.38]
$\theta_{1,0}$	Normal	0.52	0.40	[-0.11,1.21]	0.59	0.40	[-0.058,1.26]
$\theta_{2,0}$	Normal	0.61	0.18	[0.31,0.92]	0.62	0.19	[0.30,0.94]

Notes: Posterior estimates of the time-invariant parameters and the initial values of the time-varying reaction coefficients for the period from 2002.1 to 2021.2. Priors are set according to the results in Table 1.

Table B.2: Posterior estimates – Krippner shadow rate series

Param.	Rhat	N_{eff}	Mean	Std	2.5%	25%	50%	75%	97.5%
π^*	1.0000	177129	1.8268	0.0903	1.6572	1.7660	1.8234	1.8843	2.0142
σ	1.0001	32882	0.4331	0.0352	0.3682	0.4093	0.4317	0.4554	0.5063
σ_{ν_1}	1.0010	2504	0.1268	0.0796	0.0511	0.0816	0.1071	0.1461	0.3239
σ_{ν_2}	1.0005	6173	0.1010	0.0350	0.0523	0.0762	0.0946	0.1185	0.1868
$\theta_{1,1999.2}$	1.0001	37739	0.6567	0.3770	-0.0449	0.3973	0.6437	0.9015	1.4329
$\theta_{1,1999.3}$	1.0001	52780	0.6358	0.3809	-0.0738	0.3759	0.6218	0.8807	1.4205
$\theta_{1,1999.4}$	1.0001	64063	0.6181	0.3885	-0.1054	0.3565	0.6032	0.8651	1.4207
$\theta_{1,2000.1}$	1.0001	61360	0.6114	0.3992	-0.1276	0.3449	0.5949	0.8603	1.4394
$\theta_{1,2000.2}$	1.0001	60528	0.6025	0.4088	-0.1533	0.3323	0.5859	0.8543	1.4501
$\theta_{1,2000.3}$	1.0002	62000	0.5936	0.4157	-0.1740	0.3214	0.5760	0.8458	1.4596
$\theta_{1,2000.4}$	1.0001	62700	0.5842	0.4209	-0.1939	0.3105	0.5655	0.8379	1.4632
$\theta_{1,2001.1}$	1.0001	61544	0.5755	0.4240	-0.2066	0.3008	0.5557	0.8305	1.4625
$\theta_{1,2001.2}$	1.0001	60632	0.5653	0.4291	-0.2273	0.2888	0.5460	0.8205	1.4632
$\theta_{1,2001.3}$	1.0001	63907	0.5541	0.4325	-0.2455	0.2762	0.5355	0.8106	1.4511
$\theta_{1,2001.4}$	1.0001	62250	0.5447	0.4330	-0.2533	0.2668	0.5256	0.8019	1.4459
$\theta_{1,2002.1}$	1.0001	74195	0.5253	0.4337	-0.2833	0.2493	0.5096	0.7847	1.4237
$\theta_{1,2002.2}$	1.0001	78044	0.5099	0.4338	-0.3064	0.2347	0.4957	0.7705	1.4042
$\theta_{1,2002.3}$	1.0001	78458	0.4957	0.4339	-0.3243	0.2232	0.4828	0.7570	1.3874
$\theta_{1,2002.4}$	1.0001	78093	0.4809	0.4329	-0.3454	0.2102	0.4691	0.7424	1.3638

$\theta_{1,2003.1}$	1.0001	79540	0.4670	0.4288	-0.3577	0.1976	0.4575	0.7277	1.3407
$\theta_{1,2003.2}$	1.0001	75129	0.4517	0.4241	-0.3705	0.1861	0.4441	0.7114	1.3099
$\theta_{1,2003.3}$	1.0001	77333	0.4431	0.4190	-0.3726	0.1790	0.4362	0.7015	1.2924
$\theta_{1,2003.4}$	1.0001	74488	0.4327	0.4211	-0.3865	0.1712	0.4257	0.6905	1.2848
$\theta_{1,2004.1}$	1.0001	76178	0.4275	0.4209	-0.3926	0.1655	0.4208	0.6837	1.2813
$\theta_{1,2004.2}$	1.0001	79934	0.4238	0.4211	-0.3909	0.1632	0.4156	0.6797	1.2788
$\theta_{1,2004.3}$	1.0001	79640	0.4205	0.4208	-0.3920	0.1609	0.4119	0.6741	1.2730
$\theta_{1,2004.4}$	1.0001	80903	0.4172	0.4190	-0.3900	0.1584	0.4089	0.6691	1.2666
$\theta_{1,2005.1}$	1.0001	81104	0.4142	0.4158	-0.3862	0.1573	0.4054	0.6633	1.2579
$\theta_{1,2005.2}$	1.0001	80050	0.4131	0.4120	-0.3753	0.1578	0.4036	0.6594	1.2539
$\theta_{1,2005.3}$	1.0001	80023	0.4115	0.4065	-0.3648	0.1582	0.4016	0.6537	1.2435
$\theta_{1,2005.4}$	1.0001	74776	0.4113	0.3990	-0.3479	0.1606	0.4011	0.6505	1.2323
$\theta_{1,2006.1}$	1.0001	59257	0.4119	0.3903	-0.3313	0.1645	0.3991	0.6464	1.2180
$\theta_{1,2006.2}$	1.0001	46531	0.4113	0.3793	-0.3078	0.1690	0.3988	0.6403	1.2036
$\theta_{1,2006.3}$	1.0001	31865	0.4136	0.3774	-0.2964	0.1729	0.3977	0.6369	1.2067
$\theta_{1,2006.4}$	1.0001	30288	0.4052	0.3725	-0.2903	0.1687	0.3899	0.6248	1.1869
$\theta_{1,2007.1}$	1.0001	28268	0.3944	0.3685	-0.2916	0.1607	0.3789	0.6106	1.1673
$\theta_{1,2007.2}$	1.0001	30300	0.3786	0.3625	-0.2962	0.1487	0.3647	0.5899	1.1351
$\theta_{1,2007.3}$	1.0001	33753	0.3607	0.3544	-0.3012	0.1376	0.3480	0.5673	1.0939
$\theta_{1,2007.4}$	1.0001	40023	0.3406	0.3429	-0.3018	0.1231	0.3305	0.5437	1.0462
$\theta_{1,2008.1}$	1.0000	43653	0.3215	0.3315	-0.3075	0.1107	0.3138	0.5204	0.9970
$\theta_{1,2008.2}$	1.0000	49818	0.3017	0.3163	-0.3034	0.0986	0.2958	0.4955	0.9426
$\theta_{1,2008.3}$	1.0000	74209	0.2749	0.2996	-0.3111	0.0809	0.2717	0.4650	0.8765
$\theta_{1,2008.4}$	1.0001	60526	0.2228	0.3026	-0.3910	0.0365	0.2292	0.4176	0.8004
$\theta_{1,2009.1}$	1.0001	30721	0.1916	0.3038	-0.4338	0.0108	0.2026	0.3879	0.7550
$\theta_{1,2009.2}$	1.0002	14661	0.1525	0.3099	-0.4973	-0.0236	0.1696	0.3523	0.7038
$\theta_{1,2009.3}$	1.0001	23283	0.1562	0.2897	-0.4526	-0.0197	0.1683	0.3474	0.6934
$\theta_{1,2009.4}$	1.0000	69869	0.1779	0.2780	-0.3903	0.0018	0.1840	0.3608	0.7109
$\theta_{1,2010.1}$	1.0000	90803	0.1902	0.2823	-0.3824	0.0117	0.1944	0.3727	0.7372
$\theta_{1,2010.2}$	1.0000	94548	0.1930	0.2908	-0.3935	0.0112	0.1963	0.3781	0.7576
$\theta_{1,2010.3}$	1.0000	97785	0.1824	0.2992	-0.4319	-0.0011	0.1888	0.3733	0.7579
$\theta_{1,2010.4}$	1.0000	73422	0.1757	0.3113	-0.4689	-0.0103	0.1838	0.3729	0.7676
$\theta_{1,2011.1}$	1.0000	79970	0.1809	0.3225	-0.4835	-0.0085	0.1891	0.3813	0.7949
$\theta_{1,2011.2}$	1.0000	87274	0.1867	0.3310	-0.4915	-0.0054	0.1941	0.3892	0.8181
$\theta_{1,2011.3}$	1.0000	78210	0.1901	0.3393	-0.5057	-0.0029	0.1981	0.3955	0.8362
$\theta_{1,2011.4}$	1.0000	109455	0.2070	0.3392	-0.4754	0.0096	0.2106	0.4084	0.8724

$\theta_{1,2012.1}$	1.0000	106654	0.2187	0.3427	-0.4585	0.0175	0.2187	0.4187	0.9008
$\theta_{1,2012.2}$	1.0000	105884	0.2215	0.3430	-0.4536	0.0191	0.2205	0.4202	0.9077
$\theta_{1,2012.3}$	1.0000	106166	0.2245	0.3402	-0.4426	0.0227	0.2225	0.4231	0.9054
$\theta_{1,2012.4}$	1.0000	103971	0.2246	0.3347	-0.4323	0.0245	0.2215	0.4219	0.8982
$\theta_{1,2013.1}$	1.0000	104175	0.2269	0.3267	-0.4115	0.0300	0.2228	0.4200	0.8862
$\theta_{1,2013.2}$	1.0000	64795	0.2382	0.3181	-0.3740	0.0422	0.2308	0.4258	0.8939
$\theta_{1,2013.3}$	1.0000	99362	0.2249	0.3065	-0.3761	0.0356	0.2218	0.4103	0.8438
$\theta_{1,2013.4}$	1.0000	38492	0.2475	0.3037	-0.3245	0.0553	0.2376	0.4256	0.8811
$\theta_{1,2014.1}$	1.0000	31247	0.2510	0.2957	-0.3042	0.0628	0.2406	0.4250	0.8719
$\theta_{1,2014.2}$	1.0000	34851	0.2435	0.2840	-0.2943	0.0608	0.2348	0.4142	0.8370
$\theta_{1,2014.3}$	1.0001	32649	0.2338	0.2743	-0.2852	0.0569	0.2253	0.4002	0.8031
$\theta_{1,2014.4}$	1.0001	78084	0.1988	0.2581	-0.3058	0.0313	0.1954	0.3631	0.7210
$\theta_{1,2015.1}$	1.0001	93561	0.1588	0.2508	-0.3439	-0.0030	0.1601	0.3224	0.6513
$\theta_{1,2015.2}$	1.0001	40345	0.1314	0.2662	-0.4187	-0.0303	0.1369	0.3028	0.6402
$\theta_{1,2015.3}$	1.0000	37740	0.2003	0.2735	-0.3194	0.0258	0.1926	0.3656	0.7635
$\theta_{1,2015.4}$	1.0001	17871	0.2316	0.2843	-0.2871	0.0493	0.2171	0.3955	0.8384
$\theta_{1,2016.1}$	1.0001	19929	0.2230	0.2806	-0.2947	0.0412	0.2099	0.3886	0.8188
$\theta_{1,2016.2}$	1.0000	62638	0.1779	0.2650	-0.3328	0.0048	0.1730	0.3436	0.7209
$\theta_{1,2016.3}$	1.0000	62593	0.1532	0.2658	-0.3642	-0.0192	0.1493	0.3202	0.6926
$\theta_{1,2016.4}$	1.0001	22540	0.0689	0.2850	-0.5286	-0.1004	0.0802	0.2524	0.5987
$\theta_{1,2017.1}$	1.0001	25549	0.0560	0.2951	-0.5666	-0.1180	0.0683	0.2455	0.6035
$\theta_{1,2017.2}$	1.0000	40306	0.0612	0.2995	-0.5660	-0.1157	0.0712	0.2519	0.6240
$\theta_{1,2017.3}$	1.0000	87358	0.0764	0.3020	-0.5419	-0.1063	0.0809	0.2663	0.6583
$\theta_{1,2017.4}$	1.0000	101482	0.0838	0.3069	-0.5338	-0.1029	0.0866	0.2745	0.6809
$\theta_{1,2018.1}$	1.0000	101244	0.0811	0.3128	-0.5507	-0.1068	0.0838	0.2747	0.6900
$\theta_{1,2018.2}$	1.0000	89478	0.1056	0.3143	-0.5062	-0.0878	0.1016	0.2936	0.7369
$\theta_{1,2018.3}$	1.0000	44096	0.1237	0.3190	-0.4802	-0.0743	0.1161	0.3098	0.7772
$\theta_{1,2018.4}$	1.0001	30090	0.1356	0.3216	-0.4602	-0.0644	0.1245	0.3180	0.8056
$\theta_{1,2019.1}$	1.0001	16835	0.1519	0.3266	-0.4333	-0.0512	0.1355	0.3302	0.8418
$\theta_{1,2019.2}$	1.0002	13141	0.1568	0.3274	-0.4210	-0.0470	0.1377	0.3331	0.8526
$\theta_{1,2019.3}$	1.0002	11100	0.1526	0.3255	-0.4160	-0.0492	0.1339	0.3251	0.8355
$\theta_{1,2019.4}$	1.0000	96088	0.0465	0.2818	-0.5212	-0.1323	0.0503	0.2279	0.5969
$\theta_{1,2020.1}$	1.0001	30377	0.0008	0.2916	-0.5994	-0.1781	0.0091	0.1903	0.5537
$\theta_{1,2020.2}$	1.0000	54261	-0.0017	0.2911	-0.5963	-0.1855	0.0041	0.1881	0.5577
$\theta_{1,2020.3}$	1.0000	64106	-0.0084	0.2962	-0.6144	-0.1961	-0.0035	0.1859	0.5647
$\theta_{1,2020.4}$	1.0000	51970	-0.0228	0.3219	-0.6900	-0.2217	-0.0137	0.1874	0.5855

$\theta_{1,2021.1}$	1.0001	29772	-0.0465	0.3480	-0.7848	-0.2548	-0.0317	0.1808	0.6008
$\theta_{1,2021.2}$	1.0001	31254	-0.0510	0.3743	-0.8482	-0.2701	-0.0340	0.1903	0.6377
$\theta_{2,1999.2}$	1.0000	146229	0.7111	0.1741	0.3743	0.5933	0.7097	0.8269	1.0582
$\theta_{2,1999.3}$	1.0000	144693	0.7035	0.1817	0.3528	0.5813	0.7009	0.8230	1.0681
$\theta_{2,1999.4}$	1.0000	138541	0.6956	0.1832	0.3418	0.5728	0.6930	0.8151	1.0648
$\theta_{2,2000.1}$	1.0000	158879	0.6777	0.1820	0.3247	0.5564	0.6755	0.7965	1.0424
$\theta_{2,2000.2}$	1.0000	164822	0.6638	0.1798	0.3142	0.5438	0.6625	0.7820	1.0204
$\theta_{2,2000.3}$	1.0000	168282	0.6497	0.1802	0.2980	0.5300	0.6488	0.7685	1.0079
$\theta_{2,2000.4}$	1.0000	165101	0.6430	0.1867	0.2778	0.5198	0.6420	0.7654	1.0143
$\theta_{2,2001.1}$	1.0000	162927	0.6334	0.1932	0.2549	0.5068	0.6322	0.7593	1.0182
$\theta_{2,2001.2}$	1.0000	158466	0.6258	0.1965	0.2400	0.4972	0.6249	0.7530	1.0175
$\theta_{2,2001.3}$	1.0000	156512	0.6176	0.1953	0.2346	0.4894	0.6169	0.7442	1.0069
$\theta_{2,2001.4}$	1.0000	146542	0.6173	0.1921	0.2434	0.4899	0.6151	0.7428	1.0023
$\theta_{2,2002.1}$	1.0000	146512	0.5857	0.2006	0.1898	0.4541	0.5855	0.7169	0.9832
$\theta_{2,2002.2}$	1.0000	138855	0.5660	0.2100	0.1453	0.4301	0.5675	0.7030	0.9772
$\theta_{2,2002.3}$	1.0000	127120	0.5470	0.2160	0.1095	0.4091	0.5499	0.6877	0.9683
$\theta_{2,2002.4}$	1.0000	116133	0.5307	0.2193	0.0855	0.3902	0.5339	0.6742	0.9567
$\theta_{2,2003.1}$	1.0000	115133	0.5222	0.2225	0.0713	0.3795	0.5250	0.6685	0.9555
$\theta_{2,2003.2}$	1.0001	102986	0.5067	0.2267	0.0456	0.3619	0.5104	0.6557	0.9449
$\theta_{2,2003.3}$	1.0001	98950	0.4999	0.2333	0.0264	0.3514	0.5042	0.6527	0.9514
$\theta_{2,2003.4}$	1.0000	97741	0.4920	0.2447	-0.0084	0.3380	0.4969	0.6514	0.9655
$\theta_{2,2004.1}$	1.0000	98318	0.4885	0.2546	-0.0338	0.3304	0.4929	0.6528	0.9819
$\theta_{2,2004.2}$	1.0000	103340	0.4852	0.2617	-0.0535	0.3243	0.4900	0.6533	0.9921
$\theta_{2,2004.3}$	1.0001	105101	0.4819	0.2676	-0.0694	0.3169	0.4869	0.6528	1.0021
$\theta_{2,2004.4}$	1.0000	103642	0.4787	0.2718	-0.0772	0.3119	0.4833	0.6514	1.0081
$\theta_{2,2005.1}$	1.0000	103703	0.4750	0.2746	-0.0864	0.3062	0.4796	0.6482	1.0107
$\theta_{2,2005.2}$	1.0000	104344	0.4717	0.2756	-0.0911	0.3029	0.4748	0.6452	1.0113
$\theta_{2,2005.3}$	1.0000	106964	0.4685	0.2753	-0.0910	0.2995	0.4709	0.6417	1.0125
$\theta_{2,2005.4}$	1.0000	107753	0.4665	0.2738	-0.0911	0.2982	0.4681	0.6386	1.0071
$\theta_{2,2006.1}$	1.0000	110993	0.4643	0.2710	-0.0849	0.2976	0.4653	0.6338	1.0019
$\theta_{2,2006.2}$	1.0000	113264	0.4625	0.2670	-0.0771	0.2978	0.4629	0.6292	0.9951
$\theta_{2,2006.3}$	1.0000	113697	0.4616	0.2628	-0.0665	0.2992	0.4608	0.6245	0.9876
$\theta_{2,2006.4}$	1.0000	116186	0.4602	0.2567	-0.0528	0.3006	0.4595	0.6195	0.9744
$\theta_{2,2007.1}$	1.0000	119459	0.4585	0.2486	-0.0355	0.3031	0.4569	0.6128	0.9597
$\theta_{2,2007.2}$	1.0000	120184	0.4592	0.2392	-0.0153	0.3084	0.4570	0.6074	0.9430
$\theta_{2,2007.3}$	1.0000	114385	0.4601	0.2276	0.0139	0.3162	0.4567	0.6004	0.9237

$\theta_{2,2007.4}$	1.0000	106835	0.4594	0.2142	0.0436	0.3225	0.4547	0.5920	0.8984
$\theta_{2,2008.1}$	1.0000	96075	0.4585	0.1979	0.0777	0.3308	0.4535	0.5813	0.8656
$\theta_{2,2008.2}$	1.0000	75313	0.4589	0.1784	0.1195	0.3415	0.4537	0.5706	0.8287
$\theta_{2,2008.3}$	1.0000	50576	0.4677	0.1596	0.1706	0.3604	0.4613	0.5677	0.8004
$\theta_{2,2008.4}$	1.0000	23777	0.4939	0.1468	0.2272	0.3944	0.4852	0.5847	0.8068
$\theta_{2,2009.1}$	1.0000	60638	0.4027	0.1153	0.1800	0.3254	0.4014	0.4787	0.6339
$\theta_{2,2009.2}$	1.0001	38197	0.2898	0.1214	0.0439	0.2105	0.2920	0.3715	0.5226
$\theta_{2,2009.3}$	1.0001	42880	0.2631	0.1450	-0.0388	0.1721	0.2683	0.3602	0.5349
$\theta_{2,2009.4}$	1.0000	60897	0.2549	0.1622	-0.0843	0.1543	0.2603	0.3628	0.5598
$\theta_{2,2010.1}$	1.0000	74563	0.2476	0.1750	-0.1173	0.1389	0.2533	0.3632	0.5782
$\theta_{2,2010.2}$	1.0001	90128	0.2448	0.1829	-0.1324	0.1302	0.2498	0.3647	0.5939
$\theta_{2,2010.3}$	1.0000	120277	0.2507	0.1904	-0.1376	0.1302	0.2539	0.3751	0.6180
$\theta_{2,2010.4}$	1.0000	136965	0.2537	0.1962	-0.1428	0.1287	0.2555	0.3812	0.6379
$\theta_{2,2011.1}$	1.0000	135338	0.2464	0.2010	-0.1565	0.1180	0.2478	0.3773	0.6399
$\theta_{2,2011.2}$	1.0000	132815	0.2359	0.2035	-0.1722	0.1046	0.2381	0.3682	0.6341
$\theta_{2,2011.3}$	1.0000	125380	0.2196	0.2066	-0.1977	0.0876	0.2225	0.3544	0.6226
$\theta_{2,2011.4}$	1.0000	138222	0.2326	0.2108	-0.1885	0.0966	0.2329	0.3696	0.6511
$\theta_{2,2012.1}$	1.0000	126169	0.2427	0.2132	-0.1761	0.1040	0.2414	0.3803	0.6703
$\theta_{2,2012.2}$	1.0000	119926	0.2352	0.2187	-0.1934	0.0928	0.2337	0.3754	0.6758
$\theta_{2,2012.3}$	1.0000	117566	0.2221	0.2223	-0.2160	0.0778	0.2214	0.3649	0.6668
$\theta_{2,2012.4}$	1.0000	115157	0.2017	0.2282	-0.2516	0.0551	0.2020	0.3485	0.6550
$\theta_{2,2013.1}$	1.0000	109163	0.1768	0.2330	-0.2902	0.0274	0.1784	0.3280	0.6342
$\theta_{2,2013.2}$	1.0000	98911	0.1672	0.2376	-0.3115	0.0154	0.1691	0.3208	0.6338
$\theta_{2,2013.3}$	1.0000	94712	0.1548	0.2397	-0.3273	0.0018	0.1564	0.3098	0.6257
$\theta_{2,2013.4}$	1.0000	82154	0.1296	0.2410	-0.3603	-0.0233	0.1335	0.2869	0.5971
$\theta_{2,2014.1}$	1.0000	74585	0.1178	0.2411	-0.3739	-0.0346	0.1216	0.2750	0.5846
$\theta_{2,2014.2}$	1.0000	75964	0.1134	0.2422	-0.3773	-0.0398	0.1173	0.2699	0.5849
$\theta_{2,2014.3}$	1.0000	77504	0.1124	0.2434	-0.3819	-0.0414	0.1160	0.2697	0.5861
$\theta_{2,2014.4}$	1.0000	84143	0.1247	0.2435	-0.3630	-0.0306	0.1261	0.2813	0.6037
$\theta_{2,2015.1}$	1.0000	84761	0.1394	0.2413	-0.3364	-0.0159	0.1377	0.2930	0.6227
$\theta_{2,2015.2}$	1.0000	80200	0.1480	0.2354	-0.3115	-0.0054	0.1452	0.2979	0.6231
$\theta_{2,2015.3}$	1.0000	77781	0.0929	0.2327	-0.3760	-0.0560	0.0957	0.2445	0.5476
$\theta_{2,2015.4}$	1.0001	63972	0.0673	0.2363	-0.4161	-0.0819	0.0723	0.2218	0.5222
$\theta_{2,2016.1}$	1.0001	70787	0.0712	0.2391	-0.4166	-0.0795	0.0757	0.2268	0.5361
$\theta_{2,2016.2}$	1.0000	88653	0.0977	0.2402	-0.3839	-0.0546	0.0992	0.2509	0.5743
$\theta_{2,2016.3}$	1.0000	95125	0.1174	0.2413	-0.3590	-0.0366	0.1163	0.2691	0.6058

$\theta_{2,2016.4}$	1.0000	83374	0.1500	0.2415	-0.3129	-0.0071	0.1441	0.2995	0.6516
$\theta_{2,2017.1}$	1.0000	72081	0.1622	0.2377	-0.2885	0.0068	0.1543	0.3085	0.6606
$\theta_{2,2017.2}$	1.0000	71811	0.1642	0.2310	-0.2736	0.0129	0.1566	0.3070	0.6496
$\theta_{2,2017.3}$	1.0000	74840	0.1575	0.2223	-0.2658	0.0118	0.1509	0.2954	0.6208
$\theta_{2,2017.4}$	1.0000	73433	0.1567	0.2146	-0.2513	0.0152	0.1502	0.2897	0.6052
$\theta_{2,2018.1}$	1.0000	58324	0.1652	0.2105	-0.2298	0.0252	0.1569	0.2949	0.6091
$\theta_{2,2018.2}$	1.0000	83011	0.1358	0.2037	-0.2516	0.0023	0.1296	0.2630	0.5598
$\theta_{2,2018.3}$	1.0000	102143	0.1160	0.2025	-0.2735	-0.0145	0.1102	0.2414	0.5367
$\theta_{2,2018.4}$	1.0000	116612	0.1029	0.2001	-0.2804	-0.0259	0.0980	0.2258	0.5210
$\theta_{2,2019.1}$	1.0000	133434	0.0865	0.1958	-0.2915	-0.0381	0.0817	0.2055	0.4917
$\theta_{2,2019.2}$	1.0000	150393	0.0695	0.1872	-0.2949	-0.0485	0.0655	0.1831	0.4560
$\theta_{2,2019.3}$	1.0000	178870	0.0484	0.1746	-0.2970	-0.0602	0.0463	0.1554	0.4034
$\theta_{2,2019.4}$	1.0000	171428	0.0146	0.1569	-0.3054	-0.0811	0.0161	0.1129	0.3231
$\theta_{2,2020.1}$	1.0000	156466	0.0009	0.1312	-0.2680	-0.0795	0.0033	0.0842	0.2567
$\theta_{2,2020.2}$	1.0000	152337	-0.0009	0.0879	-0.1759	-0.0583	-0.0005	0.0567	0.1724
$\theta_{2,2020.3}$	1.0000	232644	-0.0061	0.0858	-0.1795	-0.0597	-0.0054	0.0482	0.1628
$\theta_{2,2020.4}$	1.0000	151244	-0.0138	0.0390	-0.0908	-0.0398	-0.0136	0.0124	0.0625
$\theta_{2,2021.1}$	1.0000	71996	0.0398	0.0925	-0.1336	-0.0211	0.0362	0.0968	0.2347
$\theta_{2,2021.2}$	1.0000	82664	0.0547	0.1004	-0.1382	-0.0122	0.0528	0.1196	0.2587

Notes: Posterior estimates of the time-invariant and the time-varying parameters obtained with the shadow rate series of Krippner for the period from 1999.2 to 2021.2. Priors are set according to the results in Table 1. $\theta_{1,1999.2}$ and $\theta_{2,1999.2}$ correspond to $\theta_{1,0}$ and $\theta_{2,0}$, respectively. N_{eff} stands for the effective sample size. Calculations carried out with ShinyStan.

Table B.3: Posterior estimates – Wu-Xia shadow rate series

Param.	Rhat	N_{eff}	Mean	Std	2.5%	25%	50%	75%	97.5%
π^*	1.0000	106351	1.8134	0.0809	1.6646	1.7594	1.8088	1.8625	1.9871
σ	1.0008	7909	0.3739	0.0417	0.2925	0.3472	0.3747	0.4015	0.4536
σ_{ν_1}	1.0021	2429	0.1352	0.0928	0.0522	0.0862	0.1153	0.1569	0.3213
σ_{ν_2}	1.0013	3646	0.1779	0.0841	0.0661	0.1125	0.1598	0.2279	0.3768
$\theta_{1,1999.2}$	1.0001	37181	0.7320	0.3736	0.0250	0.4759	0.7225	0.9769	1.4921
$\theta_{1,1999.3}$	1.0001	48261	0.7089	0.3790	-0.0061	0.4513	0.6998	0.9555	1.4823
$\theta_{1,1999.4}$	1.0001	45917	0.6906	0.3897	-0.0459	0.4297	0.6812	0.9418	1.4823
$\theta_{1,2000.1}$	1.0001	50524	0.6879	0.4016	-0.0685	0.4209	0.6769	0.9429	1.5082
$\theta_{1,2000.2}$	1.0001	50198	0.6805	0.4145	-0.0957	0.4078	0.6682	0.9400	1.5257
$\theta_{1,2000.3}$	1.0000	49685	0.6733	0.4240	-0.1174	0.3969	0.6600	0.9352	1.5415

$\theta_{1,2000.4}$	1.0000	48523	0.6666	0.4285	-0.1337	0.3880	0.6525	0.9304	1.5487
$\theta_{1,2001.1}$	1.0000	45267	0.6613	0.4310	-0.1419	0.3786	0.6454	0.9249	1.5540
$\theta_{1,2001.2}$	1.0001	42741	0.6539	0.4386	-0.1634	0.3694	0.6364	0.9199	1.5586
$\theta_{1,2001.3}$	1.0001	43181	0.6443	0.4448	-0.1818	0.3588	0.6274	0.9110	1.5616
$\theta_{1,2001.4}$	1.0001	36498	0.6378	0.4488	-0.1950	0.3506	0.6203	0.9042	1.5632
$\theta_{1,2002.1}$	1.0001	47023	0.6144	0.4494	-0.2322	0.3312	0.6016	0.8847	1.5353
$\theta_{1,2002.2}$	1.0001	47843	0.5993	0.4501	-0.2565	0.3180	0.5885	0.8725	1.5139
$\theta_{1,2002.3}$	1.0001	45666	0.5865	0.4491	-0.2724	0.3048	0.5771	0.8587	1.4984
$\theta_{1,2002.4}$	1.0001	44535	0.5731	0.4498	-0.2907	0.2932	0.5654	0.8454	1.4782
$\theta_{1,2003.1}$	1.0001	43988	0.5622	0.4471	-0.3010	0.2838	0.5546	0.8342	1.4621
$\theta_{1,2003.2}$	1.0001	37291	0.5452	0.4410	-0.3203	0.2704	0.5408	0.8168	1.4313
$\theta_{1,2003.3}$	1.0001	38693	0.5391	0.4345	-0.3109	0.2659	0.5333	0.8084	1.4168
$\theta_{1,2003.4}$	1.0001	32654	0.5269	0.4361	-0.3354	0.2559	0.5236	0.7981	1.3973
$\theta_{1,2004.1}$	1.0001	33639	0.5236	0.4348	-0.3382	0.2532	0.5201	0.7932	1.3934
$\theta_{1,2004.2}$	1.0001	33920	0.5215	0.4343	-0.3353	0.2530	0.5179	0.7888	1.3929
$\theta_{1,2004.3}$	1.0001	35824	0.5205	0.4342	-0.3309	0.2537	0.5174	0.7863	1.3922
$\theta_{1,2004.4}$	1.0001	36681	0.5204	0.4319	-0.3223	0.2539	0.5165	0.7838	1.3885
$\theta_{1,2005.1}$	1.0001	37412	0.5201	0.4257	-0.3064	0.2558	0.5155	0.7806	1.3812
$\theta_{1,2005.2}$	1.0002	36843	0.5205	0.4214	-0.2972	0.2598	0.5154	0.7770	1.3725
$\theta_{1,2005.3}$	1.0001	36379	0.5217	0.4164	-0.2895	0.2643	0.5162	0.7754	1.3627
$\theta_{1,2005.4}$	1.0001	36617	0.5255	0.4078	-0.2606	0.2707	0.5192	0.7744	1.3545
$\theta_{1,2006.1}$	1.0002	34089	0.5308	0.3979	-0.2313	0.2782	0.5233	0.7737	1.3475
$\theta_{1,2006.2}$	1.0002	29449	0.5334	0.3846	-0.2040	0.2868	0.5242	0.7695	1.3232
$\theta_{1,2006.3}$	1.0003	17832	0.5433	0.3877	-0.1808	0.2955	0.5298	0.7738	1.3510
$\theta_{1,2006.4}$	1.0003	17602	0.5364	0.3830	-0.1760	0.2910	0.5225	0.7628	1.3350
$\theta_{1,2007.1}$	1.0003	15568	0.5279	0.3825	-0.1742	0.2852	0.5136	0.7497	1.3200
$\theta_{1,2007.2}$	1.0003	15090	0.5131	0.3819	-0.1853	0.2733	0.4977	0.7301	1.2963
$\theta_{1,2007.3}$	1.0003	17072	0.4942	0.3744	-0.1920	0.2595	0.4801	0.7085	1.2588
$\theta_{1,2007.4}$	1.0002	17975	0.4743	0.3648	-0.1952	0.2452	0.4619	0.6850	1.2153
$\theta_{1,2008.1}$	1.0003	14931	0.4586	0.3594	-0.1955	0.2328	0.4463	0.6630	1.1805
$\theta_{1,2008.2}$	1.0004	12644	0.4417	0.3506	-0.1946	0.2227	0.4283	0.6405	1.1440
$\theta_{1,2008.3}$	1.0004	13502	0.4120	0.3312	-0.1996	0.2012	0.4015	0.6070	1.0791
$\theta_{1,2008.4}$	1.0002	25583	0.3254	0.3211	-0.3149	0.1276	0.3318	0.5313	0.9347
$\theta_{1,2009.1}$	1.0003	10997	0.2661	0.3386	-0.4133	0.0753	0.2830	0.4813	0.8630
$\theta_{1,2009.2}$	1.0005	6794	0.2056	0.3626	-0.5274	0.0179	0.2325	0.4318	0.8034
$\theta_{1,2009.3}$	1.0002	15755	0.2281	0.3027	-0.4043	0.0379	0.2384	0.4313	0.7950

$\theta_{1,2009.4}$	1.0003	10922	0.1981	0.3124	-0.4673	0.0088	0.2129	0.4069	0.7678
$\theta_{1,2010.1}$	1.0001	22022	0.2328	0.3013	-0.3894	0.0432	0.2415	0.4320	0.8035
$\theta_{1,2010.2}$	1.0001	26573	0.2440	0.3082	-0.3920	0.0517	0.2528	0.4452	0.8267
$\theta_{1,2010.3}$	1.0002	22773	0.2391	0.3197	-0.4376	0.0458	0.2519	0.4480	0.8312
$\theta_{1,2010.4}$	1.0001	34419	0.2581	0.3244	-0.4209	0.0621	0.2690	0.4679	0.8669
$\theta_{1,2011.1}$	1.0000	40373	0.2820	0.3372	-0.4045	0.0802	0.2892	0.4917	0.9174
$\theta_{1,2011.2}$	1.0001	31532	0.3040	0.3489	-0.3913	0.0963	0.3071	0.5124	0.9621
$\theta_{1,2011.3}$	1.0001	27986	0.3217	0.3566	-0.3761	0.1119	0.3219	0.5302	0.9990
$\theta_{1,2011.4}$	1.0002	13599	0.3514	0.3758	-0.3405	0.1344	0.3429	0.5532	1.0617
$\theta_{1,2012.1}$	1.0001	21397	0.3515	0.3677	-0.3490	0.1359	0.3456	0.5579	1.0610
$\theta_{1,2012.2}$	1.0001	34389	0.3487	0.3656	-0.3581	0.1337	0.3460	0.5583	1.0484
$\theta_{1,2012.3}$	1.0000	75479	0.3431	0.3559	-0.3687	0.1334	0.3451	0.5555	1.0310
$\theta_{1,2012.4}$	1.0001	61872	0.3378	0.3532	-0.3766	0.1319	0.3438	0.5528	1.0155
$\theta_{1,2013.1}$	1.0002	25032	0.3317	0.3541	-0.3892	0.1303	0.3412	0.5488	0.9949
$\theta_{1,2013.2}$	1.0004	14770	0.3307	0.3552	-0.3892	0.1348	0.3425	0.5486	0.9838
$\theta_{1,2013.3}$	1.0001	44546	0.3663	0.3278	-0.2923	0.1661	0.3672	0.5713	1.0125
$\theta_{1,2013.4}$	1.0000	66205	0.3996	0.3190	-0.2244	0.1989	0.3960	0.5961	1.0478
$\theta_{1,2014.1}$	1.0000	59247	0.4343	0.3122	-0.1600	0.2331	0.4251	0.6252	1.0815
$\theta_{1,2014.2}$	1.0001	47350	0.4563	0.3052	-0.1198	0.2578	0.4460	0.6434	1.0924
$\theta_{1,2014.3}$	1.0001	31929	0.4831	0.2988	-0.0724	0.2851	0.4697	0.6658	1.1146
$\theta_{1,2014.4}$	1.0001	30395	0.4898	0.2871	-0.0464	0.2978	0.4775	0.6691	1.0950
$\theta_{1,2015.1}$	1.0001	38312	0.4775	0.2787	-0.0486	0.2917	0.4679	0.6535	1.0548
$\theta_{1,2015.2}$	1.0001	43864	0.4333	0.2937	-0.1371	0.2472	0.4299	0.6167	1.0233
$\theta_{1,2015.3}$	1.0001	51282	0.4496	0.3030	-0.1266	0.2565	0.4427	0.6370	1.0707
$\theta_{1,2015.4}$	1.0001	36582	0.4791	0.3042	-0.0896	0.2800	0.4664	0.6645	1.1204
$\theta_{1,2016.1}$	1.0002	21982	0.4981	0.3062	-0.0616	0.2955	0.4812	0.6810	1.1519
$\theta_{1,2016.2}$	1.0001	40801	0.4691	0.2917	-0.0822	0.2761	0.4590	0.6509	1.0746
$\theta_{1,2016.3}$	1.0001	29681	0.4757	0.2907	-0.0637	0.2817	0.4636	0.6565	1.0877
$\theta_{1,2016.4}$	1.0001	39819	0.4378	0.2952	-0.1282	0.2471	0.4308	0.6221	1.0403
$\theta_{1,2017.1}$	1.0004	14305	0.4786	0.3246	-0.0962	0.2668	0.4572	0.6632	1.1754
$\theta_{1,2017.2}$	1.0001	43224	0.4433	0.3207	-0.1497	0.2347	0.4284	0.6354	1.1227
$\theta_{1,2017.3}$	1.0000	52662	0.4113	0.3277	-0.2086	0.2046	0.4013	0.6093	1.0859
$\theta_{1,2017.4}$	1.0001	47454	0.3944	0.3375	-0.2419	0.1867	0.3860	0.5957	1.0807
$\theta_{1,2018.1}$	1.0001	54230	0.4002	0.3408	-0.2311	0.1864	0.3881	0.5997	1.1079
$\theta_{1,2018.2}$	1.0003	17909	0.4212	0.3567	-0.2023	0.1965	0.3987	0.6145	1.1782
$\theta_{1,2018.3}$	1.0003	30061	0.4007	0.3416	-0.2208	0.1824	0.3828	0.5959	1.1278

$\theta_{1,2018.4}$	1.0001	45912	0.3778	0.3323	-0.2394	0.1654	0.3639	0.5736	1.0763
$\theta_{1,2019.1}$	1.0002	39211	0.3660	0.3226	-0.2333	0.1567	0.3529	0.5584	1.0458
$\theta_{1,2019.2}$	1.0004	17481	0.3596	0.3250	-0.2305	0.1491	0.3425	0.5473	1.0487
$\theta_{1,2019.3}$	1.0005	16138	0.3312	0.3155	-0.2446	0.1258	0.3156	0.5169	0.9950
$\theta_{1,2019.4}$	1.0001	23672	0.2276	0.3032	-0.3753	0.0357	0.2278	0.4223	0.8267
$\theta_{1,2020.1}$	1.0001	33699	0.2047	0.3050	-0.3962	0.0058	0.2030	0.4031	0.8126
$\theta_{1,2020.2}$	1.0001	25593	0.1667	0.3246	-0.4759	-0.0418	0.1673	0.3765	0.8065
$\theta_{1,2020.3}$	1.0001	19504	0.1295	0.3362	-0.5474	-0.0871	0.1332	0.3493	0.7865
$\theta_{1,2020.4}$	1.0003	11003	0.0755	0.3890	-0.7410	-0.1539	0.0932	0.3275	0.7922
$\theta_{1,2021.1}$	1.0006	6212	0.0072	0.4638	-0.9920	-0.2312	0.0464	0.3003	0.7824
$\theta_{1,2021.2}$	1.0004	8614	0.0225	0.4687	-0.9988	-0.2283	0.0591	0.3207	0.8290
$\theta_{2,1999.2}$	1.0000	134485	0.7416	0.1886	0.3774	0.6135	0.7399	0.8672	1.1172
$\theta_{2,1999.3}$	1.0000	104600	0.7423	0.2276	0.3098	0.5934	0.7359	0.8838	1.2135
$\theta_{2,1999.4}$	1.0000	75155	0.7389	0.2299	0.3040	0.5876	0.7309	0.8812	1.2208
$\theta_{2,2000.1}$	1.0000	211196	0.6920	0.2208	0.2572	0.5490	0.6901	0.8334	1.1362
$\theta_{2,2000.2}$	1.0000	217235	0.6655	0.2088	0.2505	0.5293	0.6660	0.8019	1.0773
$\theta_{2,2000.3}$	1.0000	171424	0.6380	0.2061	0.2235	0.5043	0.6404	0.7740	1.0399
$\theta_{2,2000.4}$	1.0000	210876	0.6406	0.2252	0.1880	0.4969	0.6415	0.7860	1.0869
$\theta_{2,2001.1}$	1.0000	204512	0.6301	0.2493	0.1229	0.4755	0.6320	0.7870	1.1226
$\theta_{2,2001.2}$	1.0000	202050	0.6283	0.2567	0.1088	0.4707	0.6295	0.7870	1.1399
$\theta_{2,2001.3}$	1.0000	196171	0.6237	0.2397	0.1441	0.4722	0.6243	0.7765	1.0996
$\theta_{2,2001.4}$	1.0000	98299	0.6543	0.2148	0.2376	0.5113	0.6508	0.7943	1.0862
$\theta_{2,2002.1}$	1.0000	91897	0.5614	0.2460	0.0510	0.4079	0.5685	0.7218	1.0323
$\theta_{2,2002.2}$	1.0000	64017	0.5224	0.2752	-0.0651	0.3579	0.5345	0.7018	1.0359
$\theta_{2,2002.3}$	1.0001	50366	0.4912	0.2837	-0.1155	0.3210	0.5056	0.6776	1.0174
$\theta_{2,2002.4}$	1.0001	42247	0.4666	0.2792	-0.1286	0.2940	0.4817	0.6532	0.9839
$\theta_{2,2003.1}$	1.0000	60263	0.4732	0.2748	-0.0997	0.3007	0.4836	0.6560	0.9917
$\theta_{2,2003.2}$	1.0001	35141	0.4372	0.2782	-0.1496	0.2620	0.4494	0.6250	0.9537
$\theta_{2,2003.3}$	1.0001	37836	0.4336	0.2922	-0.1821	0.2510	0.4481	0.6306	0.9748
$\theta_{2,2003.4}$	1.0001	32966	0.4151	0.3336	-0.3126	0.2172	0.4375	0.6360	1.0218
$\theta_{2,2004.1}$	1.0001	37623	0.4148	0.3688	-0.4009	0.2044	0.4407	0.6542	1.0835
$\theta_{2,2004.2}$	1.0001	40650	0.4162	0.3939	-0.4571	0.1962	0.4446	0.6654	1.1375
$\theta_{2,2004.3}$	1.0001	42209	0.4178	0.4116	-0.4988	0.1901	0.4466	0.6753	1.1715
$\theta_{2,2004.4}$	1.0001	43831	0.4186	0.4254	-0.5339	0.1865	0.4489	0.6824	1.2006
$\theta_{2,2005.1}$	1.0001	43769	0.4202	0.4331	-0.5479	0.1839	0.4516	0.6876	1.2183
$\theta_{2,2005.2}$	1.0001	43576	0.4215	0.4354	-0.5520	0.1848	0.4527	0.6889	1.2275

$\theta_{2,2005.3}$	1.0001	43789	0.4228	0.4331	-0.5452	0.1851	0.4542	0.6897	1.2247
$\theta_{2,2005.4}$	1.0001	45819	0.4282	0.4305	-0.5338	0.1919	0.4575	0.6940	1.2282
$\theta_{2,2006.1}$	1.0001	47740	0.4357	0.4242	-0.5144	0.2019	0.4634	0.6967	1.2287
$\theta_{2,2006.2}$	1.0001	48959	0.4427	0.4123	-0.4746	0.2154	0.4680	0.6985	1.2162
$\theta_{2,2006.3}$	1.0001	54323	0.4544	0.4099	-0.4556	0.2299	0.4775	0.7052	1.2296
$\theta_{2,2006.4}$	1.0000	58506	0.4652	0.4023	-0.4241	0.2458	0.4861	0.7095	1.2278
$\theta_{2,2007.1}$	1.0000	65032	0.4763	0.3894	-0.3855	0.2628	0.4949	0.7120	1.2170
$\theta_{2,2007.2}$	1.0000	75788	0.4945	0.3726	-0.3226	0.2889	0.5070	0.7195	1.2121
$\theta_{2,2007.3}$	1.0000	87931	0.5133	0.3482	-0.2393	0.3181	0.5225	0.7220	1.1913
$\theta_{2,2007.4}$	1.0001	97061	0.5279	0.3224	-0.1560	0.3437	0.5334	0.7208	1.1651
$\theta_{2,2008.1}$	1.0001	97130	0.5416	0.2859	-0.0554	0.3745	0.5433	0.7150	1.1131
$\theta_{2,2008.2}$	1.0001	86381	0.5590	0.2421	0.0674	0.4110	0.5579	0.7089	1.0473
$\theta_{2,2008.3}$	1.0002	46471	0.6105	0.2068	0.2197	0.4758	0.6022	0.7373	1.0459
$\theta_{2,2008.4}$	1.0011	5826	0.7914	0.2362	0.4055	0.6173	0.7616	0.9433	1.3100
$\theta_{2,2009.1}$	1.0002	34070	0.5395	0.1207	0.3090	0.4586	0.5374	0.6175	0.7832
$\theta_{2,2009.2}$	1.0005	6969	0.2784	0.1631	-0.0575	0.1703	0.2883	0.3927	0.5711
$\theta_{2,2009.3}$	1.0001	51153	0.4426	0.1761	0.1099	0.3278	0.4366	0.5498	0.8153
$\theta_{2,2009.4}$	1.0002	22363	0.5062	0.2364	0.0998	0.3515	0.4827	0.6328	1.0571
$\theta_{2,2010.1}$	1.0001	56559	0.4634	0.2404	0.0176	0.3094	0.4494	0.6023	0.9895
$\theta_{2,2010.2}$	1.0000	73793	0.4501	0.2447	-0.0130	0.2924	0.4395	0.5974	0.9729
$\theta_{2,2010.3}$	1.0001	38772	0.4732	0.2642	-0.0091	0.2999	0.4559	0.6280	1.0530
$\theta_{2,2010.4}$	1.0000	60858	0.4473	0.2650	-0.0478	0.2750	0.4339	0.6054	1.0154
$\theta_{2,2011.1}$	1.0000	124378	0.4102	0.2634	-0.0977	0.2402	0.4035	0.5725	0.9580
$\theta_{2,2011.2}$	1.0000	105450	0.4034	0.2582	-0.0904	0.2341	0.3961	0.5639	0.9397
$\theta_{2,2011.3}$	1.0000	135419	0.3757	0.2560	-0.1257	0.2104	0.3715	0.5370	0.8970
$\theta_{2,2011.4}$	1.0000	42287	0.4163	0.2773	-0.0943	0.2331	0.4015	0.5831	1.0128
$\theta_{2,2012.1}$	1.0001	36341	0.4058	0.2718	-0.0986	0.2248	0.3923	0.5734	0.9840
$\theta_{2,2012.2}$	1.0000	55327	0.3637	0.2805	-0.1628	0.1793	0.3522	0.5354	0.9561
$\theta_{2,2012.3}$	1.0000	140767	0.2886	0.2776	-0.2550	0.1089	0.2857	0.4645	0.8533
$\theta_{2,2012.4}$	1.0000	115604	0.2152	0.2929	-0.3847	0.0325	0.2201	0.4032	0.7869
$\theta_{2,2013.1}$	1.0001	62211	0.1542	0.3064	-0.4914	-0.0314	0.1664	0.3537	0.7330
$\theta_{2,2013.2}$	1.0001	54097	0.1202	0.3202	-0.5633	-0.0725	0.1348	0.3282	0.7203
$\theta_{2,2013.3}$	1.0002	55289	0.0970	0.3195	-0.5811	-0.0959	0.1108	0.3050	0.6976
$\theta_{2,2013.4}$	1.0002	49796	0.0715	0.3108	-0.5802	-0.1198	0.0833	0.2751	0.6568
$\theta_{2,2014.1}$	1.0002	38098	0.0380	0.3075	-0.6071	-0.1532	0.0507	0.2423	0.6168
$\theta_{2,2014.2}$	1.0002	37186	0.0248	0.3121	-0.6288	-0.1693	0.0363	0.2306	0.6145

$\theta_{2,2014.3}$	1.0002	32137	0.0011	0.3250	-0.6875	-0.1978	0.0157	0.2144	0.6082
$\theta_{2,2014.4}$	1.0001	37364	0.0109	0.3316	-0.6840	-0.1905	0.0231	0.2252	0.6411
$\theta_{2,2015.1}$	1.0001	55899	0.0453	0.3195	-0.6054	-0.1541	0.0485	0.2478	0.6743
$\theta_{2,2015.2}$	1.0001	45748	0.1189	0.2956	-0.4448	-0.0751	0.1098	0.3042	0.7318
$\theta_{2,2015.3}$	1.0001	57731	0.0675	0.2812	-0.4816	-0.1164	0.0636	0.2479	0.6371
$\theta_{2,2015.4}$	1.0001	44040	-0.0204	0.2905	-0.6048	-0.2067	-0.0165	0.1691	0.5470
$\theta_{2,2016.1}$	1.0002	25878	-0.0875	0.3147	-0.7466	-0.2839	-0.0745	0.1212	0.5062
$\theta_{2,2016.2}$	1.0001	40862	-0.0677	0.3275	-0.7435	-0.2706	-0.0598	0.1421	0.5681
$\theta_{2,2016.3}$	1.0001	38333	-0.0924	0.3439	-0.8185	-0.2989	-0.0786	0.1266	0.5661
$\theta_{2,2016.4}$	1.0001	50423	-0.0842	0.3393	-0.7930	-0.2885	-0.0749	0.1296	0.5728
$\theta_{2,2017.1}$	1.0002	27934	-0.1419	0.3343	-0.8644	-0.3408	-0.1217	0.0772	0.4741
$\theta_{2,2017.2}$	1.0001	54409	-0.0873	0.3027	-0.7086	-0.2759	-0.0811	0.1070	0.5009
$\theta_{2,2017.3}$	1.0001	74176	-0.0363	0.2825	-0.5862	-0.2196	-0.0396	0.1402	0.5430
$\theta_{2,2017.4}$	1.0001	67681	-0.0249	0.2673	-0.5374	-0.1995	-0.0307	0.1415	0.5266
$\theta_{2,2018.1}$	1.0000	90295	-0.0853	0.2497	-0.5839	-0.2473	-0.0841	0.0778	0.4070
$\theta_{2,2018.2}$	1.0003	17573	-0.2057	0.2746	-0.7969	-0.3763	-0.1877	-0.0172	0.2888
$\theta_{2,2018.3}$	1.0001	30297	-0.1748	0.2801	-0.7788	-0.3438	-0.1573	0.0121	0.3364
$\theta_{2,2018.4}$	1.0001	70048	-0.1324	0.2856	-0.7368	-0.3020	-0.1209	0.0489	0.4140
$\theta_{2,2019.1}$	1.0001	91339	-0.1186	0.2950	-0.7451	-0.2888	-0.1068	0.0619	0.4540
$\theta_{2,2019.2}$	1.0000	113266	-0.1047	0.2900	-0.7218	-0.2698	-0.0950	0.0694	0.4636
$\theta_{2,2019.3}$	1.0001	73503	-0.1176	0.2802	-0.7277	-0.2718	-0.1020	0.0515	0.4137
$\theta_{2,2019.4}$	1.0002	23663	-0.1628	0.2673	-0.7897	-0.2990	-0.1322	0.0058	0.2914
$\theta_{2,2020.1}$	1.0001	40890	-0.1171	0.2118	-0.5991	-0.2302	-0.0996	0.0138	0.2631
$\theta_{2,2020.2}$	1.0000	46988	-0.0778	0.0987	-0.2752	-0.1429	-0.0763	-0.0119	0.1131
$\theta_{2,2020.3}$	1.0000	87838	-0.0542	0.1262	-0.3222	-0.1274	-0.0500	0.0231	0.1915
$\theta_{2,2020.4}$	1.0001	31429	-0.0321	0.0378	-0.1082	-0.0567	-0.0315	-0.0068	0.0402
$\theta_{2,2021.1}$	1.0008	5198	0.2557	0.2018	-0.0530	0.1047	0.2234	0.3820	0.7123
$\theta_{2,2021.2}$	1.0001	19628	0.0971	0.1121	-0.1243	0.0245	0.0977	0.1712	0.3141

Notes: Posterior estimates of the time-invariant and the time-varying parameters obtained with the shadow rate series of Wu and Xia for the period from 1999.2 to 2021.2. Priors are set according to the results in Table 1. $\theta_{1,1999.2}$ and $\theta_{2,1999.2}$ correspond to $\theta_{1,0}$ and $\theta_{2,0}$, respectively. N_{eff} stands for the effective sample size. Calculations carried out with ShinyStan.

Table B.4: Posterior distribution estimates with tight priors – 1999.2-2021.2

		Krippner			Wu-Xia		
Param.	Density	Mean	Std	[0.05, 0.95]	Mean	Std	[0.05, 0.95]
π^*	Normal	1.84	0.93	[1.69,2.00]	1.83	0.087	[1.79,1.98]
σ	InvGamma	0.44	0.033	[0.39,0.50]	0.40	0.036	[0.34,0.45]
σ_{ν_1}	InvGamma	0.083	0.034	[0.044,0.15]	0.090	0.038	[0.046,0.16]
σ_{ν_2}	InvGamma	0.076	0.025	[0.045,0.12]	0.11	0.056	[0.054,0.23]
$\theta_{1,0}$	Normal	0.57	0.30	[0.086,1.08]	0.63	0.31	[0.14,1.14]
$\theta_{2,0}$	Normal	0.71	0.14	[0.48,0.94]	0.73	0.15	[0.49,0.97]

Notes: Posterior distribution estimates for time-invariant parameters for the period from 1999.2 to 2021.2 with tight priors for the standard deviations of the innovations in the processes governing the time-varying response to the expected inflation gap and the expected growth gap. The sample period from 1999.2 to 2001.4 was used as a training sample. The results obtained from the training sample were used as priors to estimate the model on the complete sample.

Table B.5: Posterior distribution estimates with tight priors and without model training – 1999.2-2021.2

		Krippner			Wu-Xia		
Param.	Density	Mean	Std	[0.05, 0.95]	Mean	Std	[0.05, 0.95]
σ	InvGamma	0.46	0.037	[0.40,0.52]	0.42	0.038	[0.36,0.48]
π^*	Normal	1.84	0.13	[1.64,2.05]	1.83	0.12	[1.66,2.04]
$\theta_{1,0}$	Normal	0.35	0.41	[-0.28,1.06]	0.44	0.43	[-0.22,1.17]
$\theta_{2,0}$	Normal	0.59	0.25	[0.18,1.00]	0.63	0.30	[0.16,1.12]
σ_{ν_1}	InvGamma	0.081	0.032	[0.043,0.14]	0.088	0.036	[0.045,0.16]
σ_{ν_2}	InvGamma	0.076	0.026	[0.044,0.12]	0.11	0.052	[0.052,0.21]

Notes: Posterior distribution estimates for time-invariant parameters for the period from 1999.2 to 2021.2 with tight priors for the standard deviations of the innovations in the processes governing the time-varying response to the expected inflation gap and the expected growth gap. The estimation was carried out without conducting any training of the model.

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